On Finding a Complete Integral of Second-Order Hyperbolic PDEs with Constant Coefficients in Infinite Space

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Abstract

In this paper we describe the process of finding a general solution of the equation

$$au_{xx} + 2bu_{xy} + cu_{yy} + du_x + fu_y + gu = 0 \ ,$$

where $a \neq 0$, b, c, d, f, g are arbitrary real constants and $b^2 - ac > 0$, using the computer algebra system *Mathematica*. Finding the solution of this equation is based on its preliminary reducing to canonical form

$$v_{\xi\eta} + a_1 v_\xi + b_1 v_\eta + c_1 v = 0 \ ,$$

where $v(\xi(x; y), \eta(x; y))$ is the unknown function expressed in transformed coordinates, a_1, b_1, c_1 are some numerical coefficients following from the coordinate transformation. The formulae for a_1, b_1, c_1 and the transformed coordinates $\xi(x, y)$, $\eta(x, y)$ will be obtained in the process of reducing to canonical form. Then the further simplifications are performed with use of substitution

$$v(\xi, \eta) = e^{\lambda\xi+\mu\eta} w(\xi, \eta) \ .$$

The appropriate values of the constants λ and μ allow the coefficients of first-order derivatives of w to be eliminated. The simplified equation thus obtained is solved in the closed form, and the general solution of the original equation is presented. Further, we discuss the application of the obtained exact solutions to some "real-world" second order hyperbolic partial differential equations.