On the computation of π-flat outputs for differential-delay systems

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A control system of linear differential equations is *flat* if all its variables can be expressed as functions of a particular output and a finite number of its derivatives. See, e. g., [2, 3] for the definition. One can translate this idea directly to the case of linear differential-delay systems. However, since only few interesting systems will be flat in this sense, in [4] for the case of constant coefficients a so-called *liberation polynomial* or *prediction* π was introduced. A system is π-*flat* if its variables are expressible by a particular output and a finite number of its derivatives, delays and predictions by π^{-1}. Computational issues have been treated, e. g., in [1].

In this contribution we present an extension of this concept to non-constant coefficients. Algebraically, we will model linear differential-delay operators by an Ore algebra $\mathcal{O} = \mathcal{K}[\delta, d/dt]$ where $\mathcal{K}$ are the meromorphic functions over the real line, d/dt is the usual derivation and δ is the delay $\delta(a(t)) = a(t-1)$. Systems are given in the form

$$Ax = Bu$$

where A and B are matrices over $\mathcal{O}$ and where x is the *state vector* of the system and u is the *input vector*.

We will present a definition of π-flatness for such systems with a fixed input-output structure where π is a delay operator in $\mathcal{K}[\delta]$. Additionally, we provide a characterisation in terms of *hyper-regularity* of the system matrices which will lead to an algorithm to compute a π-flat output if it exists. We will compare our flatness definition with the classical definition in the case of systems with constant coefficients and without a given input-output structure. Finally, we will discuss the action of π^{-1} on the signal space.

References

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