Composition and Factorization of Generalized Inverses and Boundary Problems

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We present ongoing work on generalizing results from [2, 3] for composing and factoring regular boundary problems to singular (overdetermined) boundary problems and generalized Green's Operators as described in [1].

First, we recall the composition and factorization algorithms for regular boundary problems (i.e. for problems having a unique solution for every right hand side), and do some sample computations in our MAPLE package *IntDiffOp*. In the generalized setting, defining a composition corresponding to the multiplication of the solution operators is more involved. In contrast to the regular case, where the Green's Operator of a boundary problem is a particular right inverse of the corresponding differential operator, generalized Green's Operators only satisfy the weaker conditions of generalized inverses, which are not closed under composition. Hence the product of generalized Green's Operators is not necessarily a generalized Green's Operator.

We give necessary and sufficient conditions for the product of generalized inverses to be a generalized inverse of the product of the original operators in terms of the defining subspaces. For finite (co-)dimensional spaces, these conditions can be checked algorithmically. Furthermore, we present some first steps towards factoring generalized boundary problems into regular and singular factors, and their realisation in the *IntDiffOp* package.

This talk is based on joint work with Georg Regensburger.

References

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