On a differential algebraic approach of control observation problems

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Abstract. Observation problems in control systems literature generally (see for instance [5, 4]) refer to problems of estimation of state variables (or identification of model parameters) from two sources of information: online measurements of some variables, and first order dynamic models relating the quantities to be estimated and those assumed being online measured. Observation problems are addressed here

(i) by allowing dynamic models to be implicit and of arbitrary order but restricted to be polynomial in variables and their derivatives,

(ii) and by considering the more general situation of estimating one subset of system variables with respect to another subset of the system variables.

Specifically, given a dynamic system described by algebraic differential equations

$$\left\{ \begin{array}{ll} P_i(w, \xi, \zeta) = 0 & (i = 1, 2, \cdots), \\ Q_j(w, \xi, \zeta) = 0 & (j = 1, 2, \cdots), \end{array} \right. \tag{1}$$

one observation problem consists of the *online* estimation of $\xi(t) \in \mathbb{R}^\nu$ from the knowledge of the P_i's and Q_j's and time histories ($[t_0, t] \ni \tau \longmapsto w(\tau) \in \mathbb{R}^\mu$) of w. Here the P_i's and Q_j's are differential polynomials in w, ξ and ζ, and ν, μ are natural integers. This problem is under investigation since the pioneering work of R. E. Kalman in the late fifties addressing its linear context. A complete nonlinear answer is still lacking. A general approach consists of a two part theory: one of *observability*, that is, derivation of conditions on the P_i's and Q_j's guaranteeing the ability to some how estimate ξ from the supposedly known data, and another part of the theory, the *observer design*, yielding algorithms for such an estimation of ξ.

Though central the previous observation problem (observability and observer design) is not the only one. For instance, closely related to it, are two problems of *robustness* with respect to model and measurements uncertainties. Another observation problem with important practical potential consists of determining subsets w of systems variables which make a given subset ξ observable.

Starting from the mid eighties (see [6, 3, 2, 1]) differential algebra and differential algebraic decision methods have been shown to provide a quite consistent language to describe some of these observation problems along with some of their solutions.

An account of this is proposed in this presentation. In particular, open problems will be described, including theoretical ones and those related to lack of efficient implementation of differential algebraic decision methods.

References

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