Algorithms for non integrabililty proofs

Thierry COMBOT

We present an algorithm that takes a family of homogeneous potentials $V(a,q)$ with parameters a and return a subset of the parameter space such that all potentials satisfy all Morales Ramis conditions of integrabililty. These rely on the Galois group of variational equations near straight line orbits, which we will be analyzed systematically up to order 2. The first order criterion necessitates that the eigenvalues of Hessian matrices along straight line orbits should belong to some infinite discrete set. This criterion is analyzed through two steps

- The construction of a comprehensive Groebner basis is avoided thanks to adding to the initial family some subfamilies with fewer straight line orbits, proving these are only the degenerated cases.
- We prove a relation on eigenvalues similar to Maciejewski Pryzbylska that allows generally to bound the admissible set of eigenvalues of Morales-Ramis.

In a specific case where this relation does not hold, we perform an analysis of higher variational equations showing that this case could only correspond to potentials invariant by rotation. To produce an effective criterion in practice, we look at order 2, which after reduction correspond to study the zeros of a 2-indexes holonomic sequence (one for the eigenvalue, the other for the homogeneity degree). The criterion is then that some third order derivatives should vanish. We eventually apply this algorithm finding all meromorphically integrable homogeneous polynomial potentials up to degree 9, and several families with singularities similar to the three-body problem.