Homomorphism between two difference operators

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Abstract

Let $D := \mathbb{C}(x)[\tau]$ be the ring of difference operators with rational function coefficients, and let $L_1, L_2 \in D$. Denote $V(L_1)$ and $V(L_2)$ as their solution spaces inside a universal extension. In this talk, we will present an algorithm that computes the set, denoted $\mathrm{Hom}(L_1, L_2)$, of difference operators $R \in D \pmod{L_1}$ for which $R(V(L_1)) \subseteq V(L_2)$.

There is one to one correspondence between $\mathrm{Hom}(L_1, L_2)$ and rational (invariant under the difference Galois group) elements of $V(L_1^*) \otimes V(L_2)$, where L_1^* is the adjoint of L_1. We define a space $\mathcal{M}(L_1^*, L_2)$ that is isomorphic to $V(L_1^*) \otimes V(L_2)$ and compute its rational elements. This is done by working directly with L_1^* and L_2, we avoid computing large operators such as the symmetric product of L_1^* and L_2 (whose solution space is a homomorphic image of $\mathcal{M}(L_1^*, L_2)$.)

(Joint work with Mark van Hoeij)