Duhamel – Type Representation of the Solution of a Linear Nonlocal Boundary Value Problem

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Consider the equation of a free supported beam:

$$\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^4 u}{\partial x^4},\ 0 < x < 1,\ 0 < t < \infty,$$

and the initial – boundary value conditions:

$$\begin{aligned} &u(0,t) = 0,\ u_{xx}(0,t) = 0 \\ &\int_0^1 u(\xi,t)d\xi = 0,\ u_x(1,t) - u_x(0,t) = 0 \\ &u(x,0) = f(x),\ u_t(x,0) = g(x) \end{aligned}$$

The way of derivation of Duhamel – type representation of the solution $u(x,t)$ using multivariate operational calculus (see [1]) and symbolic manipulations by means of the computer algebra system *Mathematica* is considered. This representation includes multivariate convolution products of a special solution satisfying simple boundary value conditions and a given boundary value function. The case when $f(x) \neq 0$, $g(x) = 0$ is considered in more details.

The obtained representation is used successfully for numerical computation and visualization of the solution.

Similar problems are considered in [2].

References

[1] I.Dimovski, *Convolutional Calculus*, Kluwer Acad. Publishers, Dordrecht, 1990.

[2] I.Dimovski, M.Spiridonova, Computational Approach to Nonlocal Boundary Value Problems by Multivariate Operational Calculus , *Math. Sci. Res. J.*, V. 9, No 12, 2005, 315-329