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Rota-Baxter Type and Differential Type Algebras

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Abstract: A Rota-Baxter k-algebra R is a (not necessarily commutative, but associative) k-algebra (where k is a unitary commutative ring) with a linear operator $P : R \to R$ satisfying the following Rota-Baxter identity for all $x, y \in R$:

$$P(x)P(y) = P(P(x)y + xP(y))$$

The identity reflects the integration-by-parts formula of calculus, but there are many other examples of Rota-Baxter algebras. A differential k-algebra R is (usually commutative and associative) k-algebra with a linear operator $\delta : R \to R$ satisfying the Leibniz identity for all $x, y \in R$:

$$\delta(xy) = \delta(x)y + x\delta(y).$$

In this talk we describe a framework to solve a still open problem of Rota: to classify all possible Rota-Baxter *type* algebras, where the Rota-Baxter identity is modified or generalized. Dually, the framework will also work for differential *type* algebras when the Leibniz identity is modified or generalized. We produce through computation 15 classes of Rota-Baxter type algebras, many of which are new. Research in the case for differential type algebras has not even begun, although some work on differential Rota-Baxter algebras has started.

Both Rota-Baxter algebras and differential algebras have wide applications such as in combinatorics, number theory, quantum field-theory and differential equations.

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