# Extensions of the Heaviside Algorithm and the Duhamel Principle for Nonlocal Cauchy Problems

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It is proposed an extension of the Heaviside algorithm to nonlocal Cauchy problems for Linear Ordinary Differential Equations (LODE) with constant coefficients based on an Operational Calculus of Mikusinski's type. A special case of this approach is used for explicit determination of periodic solutions of LODE both in the non-resonance and the resonance cases.

We consider nonlocal Cauchy problems consisting in solution of LODE with constant coefficients

$$P\left(\frac{d}{dt}\right) y \;=\; F(t)$$

under the "initial" conditions

$$\Phi\{\, y^{(k)}\} \;=\; 0,\;\; k \;=\; 1,\, 1,\, 2,\, \ldots,\, degP - 1,$$

where $F(t) \in C(I\!\!R)$ and $\Phi$ is a linear functional in $C(I\!\!R)$.

The convolution

$$(f * g)(t) \;=\; \Phi_\tau \{ \int_\tau^t f(t + \tau - \sigma)\, g(\sigma)\, d\sigma \},$$

introduced by one of the authors in 1974 (see [1]) allows to build a Mikusinski's type operational calculus based on it.

We use an extension of the Heaviside algorithm (see [2]) for obtaining of the special solution for the choice $F(t) \;\equiv\; 1$ only. It is combined with the Duhamel principle in the following way: let Y= Y(t) be the solution of the nonlocal Cauchy problem for $F(t) \;\equiv\; 1$; then the solution for arbitrary $F(t)$ is given by

$$y(t) \;=\; \frac{d}{dt}\, \Phi_\tau \{ \int_\tau^t Y(t + \tau - \sigma)\, F(\sigma)\, d\sigma \}.$$

This approach specialized to the functional $\Phi\{f\} \;=\; \frac{1}{T} \int_0^T f(\tau)\, d\tau$ allows to propose an efficient algorithm for obtaining of the periodic solutions of LODE with constant coefficients both in the non-resonance and in the resonance cases (see [3]). This algorithm was implemented and experimented using the computer algebra system *Mathematica*.

**References.** [1] I. H. Dimovski. Convolutional Calculus. Kluwer, Dordrecht, 1990. [2] I. H. Dimovski. Nonlocal operational calculi. In Proc. Steklov Inst. of Math., 1995, Issue 3, 53-65. [3] S. I. Grozdev. A convolutional approach to initial value problems for equations with right invertible operators. Compt. Rend., Bulg. Acad. of Sci., 33, 1 (1980), 35-38.