Extensions of the Heaviside Algorithm
and the Duhamel Principle for Nonlocal Cauchy Problems

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It is proposed an extension of the Heaviside algorithm to nonlocal Cauchy problems for Linear Ordinary Differential Equations (LODE) with constant coefficients based on an Operational Calculus of Mikusinski’s type. A special case of this approach is used for explicit determination of periodic solutions of LODE both in the non-resonance and the resonance cases.

We consider nonlocal Cauchy problems consisting in solution of LODE with constant coefficients

\[ P \left( \frac{d}{dt} \right) y = F(t) \]

under the “initial” conditions

\[ \Phi \{ y^{(k)} \} = 0, \ k = 1, 1, 2, \ldots, \deg P - 1, \]

where \( F(t) \in C(I \mathbb{R}) \) and \( \Phi \) is a linear functional in \( C(I \mathbb{R}) \).

The convolution

\[ (f \ast g)(t) = \Phi_\tau \left\{ \int_\tau^t f(t + \tau - \sigma) g(\sigma) d\sigma \right\}, \]

introduced by one of the authors in 1974 (see [1]) allows to build a Mikusinski’s type operational calculus based on it.

We use an extension of the Heaviside algorithm (see [2]) for obtaining of the special solution for the choice \( F(t) \equiv 1 \) only. It is combined with the Duhamel principle in the following way: let \( Y = Y(t) \) be the solution of the nonlocal Cauchy problem for \( F(t) \equiv 1 \); then the solution for arbitrary \( F(t) \) is given by

\[ y(t) = \frac{d}{dt} \Phi_\tau \left\{ \int_\tau^t Y(t + \tau - \sigma) F(\sigma) d\sigma \right\}. \]

This approach specialized to the functional \( \Phi \{ f \} = \frac{1}{T} \int_0^T f(\tau) d\tau \) allows to propose an efficient algorithm for obtaining of the periodic solutions of LODE with constant coefficients both in the non-resonance and in the resonance cases (see [3]). This algorithm was implemented and experimented using the computer algebra system Mathematica.