



Special Issue on Applications of Gröbner Bases Foreword of the Guest Editors

The Gröbner basis method was introduced in 1965 by Buchberger. In the last three and a half decades work by the inventor and other researchers has added generalizations and many more applications of the method. The Gröbner basis method has become one of the most important techniques in providing exact solutions of nonlinear problems in multivariate polynomial ideal theory, in computational commutative algebra, in elimination theory, in solving systems of algebraic equations, and in many other related areas. It is also being used fruitfully in a variety of seemingly unrelated research areas such as geometrical theorem proving, integer programming, solid modeling and engineering. The method is implemented in all major computer algebra systems.

Nevertheless, the field is still under active development both in the direction of improving the method by new theoretical insights and in finding new applications. This special issue is dedicated to reporting serious applications of the Gröbner basis methods to mathematics, science, engineering, education and other areas.

Among twenty one submissions, we have chosen seven papers to appear in this special issue. We give a brief summary of each paper.

The paper “Computing Ideals of Points,” by J. Abbott, A. Bigatti, M. Kreuzer and L. Robbiano, draws on earlier work on the ideals of points in affine and projective space but makes two new contributions. The first is a modular version of the Buchberger-Möller algorithm, and the second is an improved algorithm for the projective case. The paper also includes some good examples, a clear complexity analysis, and a discussion of implementation issues and timings.

In the paper “Reduced Gröbner Bases of Free Difference-Differential Modules and Difference-Differential Dimension Polynomials” Levin introduces a special term ordering for linear difference-differential equations such that a dimension polynomial in two variables (one for the differential operators and one for the difference operators) may be obtained algorithmically via Gröbner-basis-type techniques. The results have applications to linear difference-differential equations since some important invariants may be found by computing the dimension polynomial.

The paper “Cellular Binomial Ideals: Primary Decomposition of Binomial Ideals,” by I. Ojeda and R. Piedra, contains interesting modifications and improvements on some of the results in the paper “Binomial Ideals” by Eisenbud and Sturmfels (see the bibliography of the paper).

In the Bodnar and Schicho paper “Automated Resolution of Singularities for Hypersurfaces,” the authors give a description of an algorithm for the resolution of singularities in the cases of hypersurfaces in characteristic zero. The algorithm, which has been implemented in MAPLE, is based on a constructive proof of Villamayor (see the bibliography of the paper).

The paper “Computing Gröbner Bases by FGLM Techniques in a Noncommutative Setting,” by M. Borges-Trenard, M. Borges-Quintana and T. Mora, adapts the meth-

ods and results developed for the commutative setting in FGLM and MMM (see the bibliography of the paper) to the noncommutative setting.

In the paper “A Fast Algorithm for Gröbner Basis Conversion and Its Applications,” Tran presents a deterministic method to vary the target point of a Gröbner walk to ensure the generality of the position, i.e., we always have just a few terms in the initial forms during the walk. The method is especially useful when the target point of the walking path lies on the intersection of many cones. The author also presents some applications of the conversion methods for implicitization and geometric reasoning.

Muller-Quade and Steinwandt in “Gröbner Bases Applied to Finitely Generated Field Extensions,” show how to determine certain properties of fields such as the transcendence degree without using tag variables.

Finally, we would like to thank all the people who have contributed to this special issue: the authors of all submitted papers and the referees.

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