



Methods for Choosing the Regularization Parameter

An Overview of our Current Research

Frank Bauer (fbauer@math.uni-goettingen.de)

Georg-August Universität Göttingen

17.11.2005

Inverse Problems

- ▶ Many domains of modern technology require the solution of unstable problems
- ▶ In order to solve these problems one has to balance
 - ▶ Measurements which we cannot really trust
 - ▶ A-priori assumptions which we do not really know

Inverse Problems

- ▶ Many domains of modern technology require the solution of unstable problems
- ▶ In order to solve these problems one has to balance
 - ▶ Measurements which we cannot really trust
 - ▶ A-priori assumptions which we do not really know

Regularization

- ▶ This balancing is done by regularization methods, e.g. Tikhonov:

$$x_{\text{sol}} = \underset{x}{\operatorname{argmin}} \quad \text{distance to measurement} + \alpha \quad \text{distance to a-priori assump.}$$

- ▶ One crucial point is finding the regularization parameter α

More Difficulties

- ▶ Noise structure badly known
- ▶ In non-classical but natural noise assumptions the energy of the noise can be infinite
- ▶ Sometimes we are interested in solutions in Non-Hilbert spaces

More Difficulties

- ▶ Noise structure badly known
- ▶ In non-classical but natural noise assumptions the energy of the noise can be infinite
- ▶ Sometimes we are interested in solutions in Non-Hilbert spaces

Selected Methods

- ▶ Cross-Validation
- ▶ L-Curve, Generalized Cross-Validation and others
- ▶ Morozov
- ▶ **Balancing principle**

Input

- ▶ Expectation of the noise with respect to the regularization parameter and measurement noise
- ▶ Regularized solutions with respect to the regularization parameters

Input

- ▶ Expectation of the noise with respect to the regularization parameter and measurement noise
- ▶ Regularized solutions with respect to the regularization parameters

Where it works (provenly)

- ▶ Linear problems with almost all regularization methods
- ▶ Non-linear problems with some regularization methods
- ▶ Classical and Stochastic noise
- ▶ Metric solution spaces
- ▶ Some Multi-parameter regularization methods

