

Inverse problems and model identification in mechanics

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Progress Report

Overview

- Projected Newton for inverse problems with constraints
- Homogenization for piezoelectric stack actuator
- Further discussions and new contacts
- Activities

Constrained Inverse Problems

joint work with Andreas Neubauer

Example: Identify $a = a(x)$ in

$$\nabla(a\nabla u) = f \text{ in } \Omega \subset \mathbb{R}^n$$

from measurements of u , under the constraint

$$\underline{\gamma} \leq a(x) \leq \bar{\gamma} \quad \forall x \in \Omega$$

\rightsquigarrow ellipticity and boundedness of $\nabla(a\nabla\cdot) : H_0^1(\Omega) \rightarrow H^{-1}(\Omega)$.

Formulation as Operator Equation in Hilbert Space

$$F(a) = y^\delta$$

$$\begin{aligned} F : \mathcal{D}(F) \subset X &\rightarrow Y \\ a &\mapsto u \text{ solution to } \nabla(a\nabla u) = f \end{aligned}$$

L^∞ measurement error, Y Hilbert space $\rightarrow Y = L^2(\Omega)$.

L^∞ bound constraints, X Hilbert space $\rightarrow X = H^{\frac{n}{2}+\varepsilon}(\Omega)$, $\mathcal{D}(F) = B_\rho^X(a_0)$

\leadsto oversmoothing of coefficient, ill-posedness is artificially increased!

alternative:

$$X = L^2(\Omega), \quad \mathcal{D}(F) = \{a \in L^2(\Omega) \mid \underline{\gamma} \leq a \leq \bar{\gamma} \text{ a.e. in } \Omega\}$$

\leadsto jumps in coefficients are possible

Regularization Methods for Constrained Inverse Problems (I)

Tikhonov-Philips:

$$\min_x \|F(x) - y^\delta\|^2 + \alpha \|x - x_0\|^2 \quad x \in \mathcal{D}(F) \quad [\text{Neubauer 88}], [\text{Engl\&Hanke\&Neubauer96}]$$

Landweber:

$$x_{k+1} = Proj_{\mathcal{D}(F)} \left(x_k - F'(x_k)^* (F(x_k) - y^\delta) \right) \quad [\text{Eicke 92}], \text{ linear case}$$

proof only goes through for special cases of $\mathcal{D}(F)$!

Levenberg-Marquardt:

$$x_{k+1} = Proj_{\mathcal{D}(F)} \left(x_k - (F'(x_k)^* F'(x_k) + \alpha_k I)^{-1} (F(x_k) - y^\delta) \right)$$

analogously to Landweber!

Iteratively regularized Gauss-Newton:

$$x_{k+1} = Proj_{\mathcal{D}(F)} \left(x_0 - (F'(x_k)^* F'(x_k) + \alpha_k I)^{-1} F'(x_k)^* \left(F(x_k) - y^\delta - F'(x_k)(x_k - x_0) \right) \right)$$

Regularization Methods for Constrained Inverse Problems (II)

Iteratively regularized Gauss-Newton:

$$x_{k+1} = Proj_{\mathcal{D}(F)} \left(x_0 - \underbrace{(F'(x_k)^* F'(x_k)) + \alpha_k I}_{\text{linear Tikhonov}}^{-1} F'(x_k)^* (F(x_k) - y^\delta - F'(x_k)(x_k - x_0)) \right)$$

- convergence and optimal convergence rates
also for linear Landweber or iterated Tikhonov in place of linear Tikhonov
- projection onto general convex closed domain $\mathcal{D}(F)$
- generalization to large class of linear regularization methods
in place of linear Tikhonov (e.g., regularization by discretization, ν -methods)
for sufficiently smooth solutions ($\mu \geq \frac{1}{2}$ in source condition)

Numerical Tests (I)

$$a(x) = \begin{cases} 1 & x \in [0, \frac{1}{2}] \\ (1 + c) & x \in (\frac{1}{2}, 1] \end{cases}$$

$$u(x) = \sin(\pi x)$$

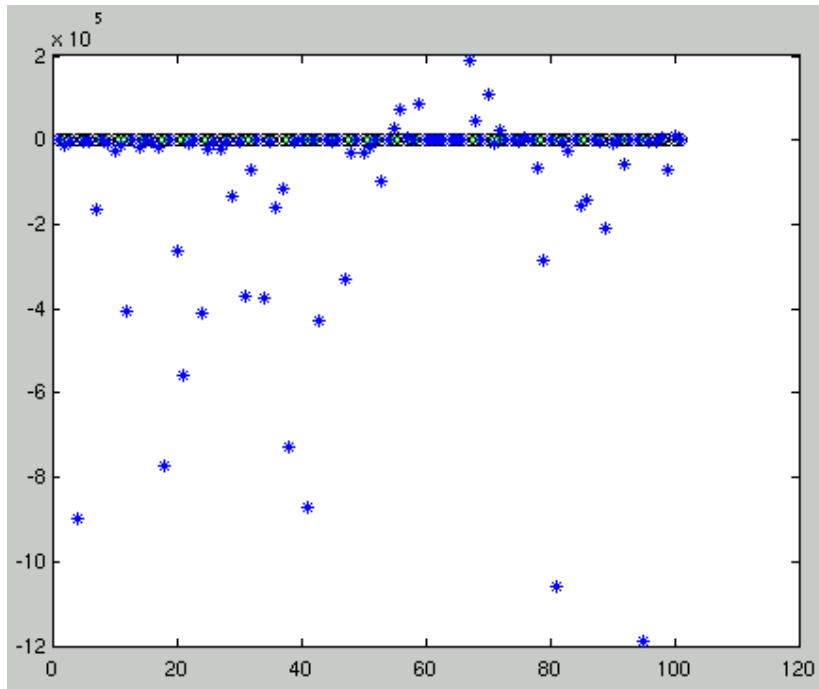
$$\underline{\gamma} = 0.1 \quad \bar{\gamma} = 25$$

$$c = 10 \text{ (height of jump)}$$

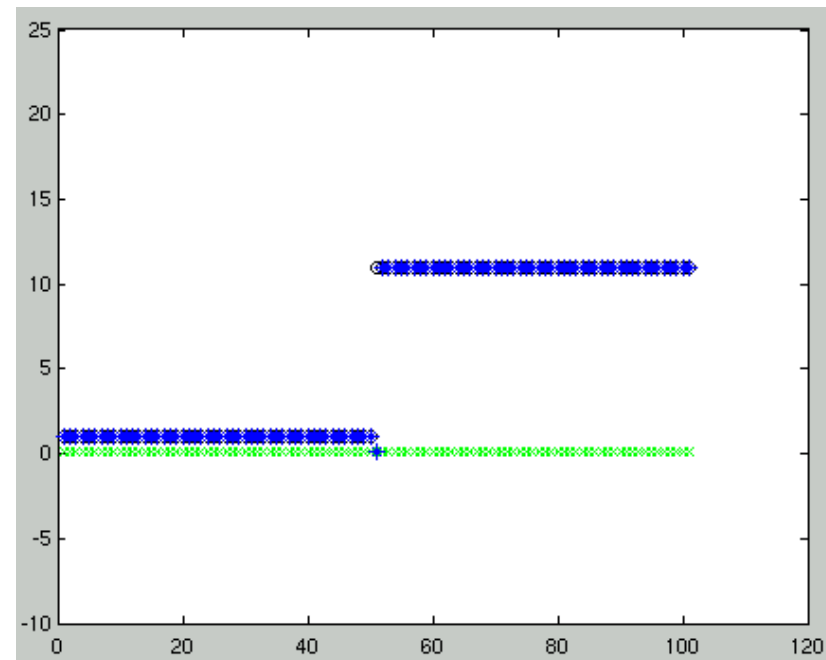
$$a_0(x) \equiv 0.1$$

Numerical Tests (II)

Gauss-Newton without projection
after 150 iterations

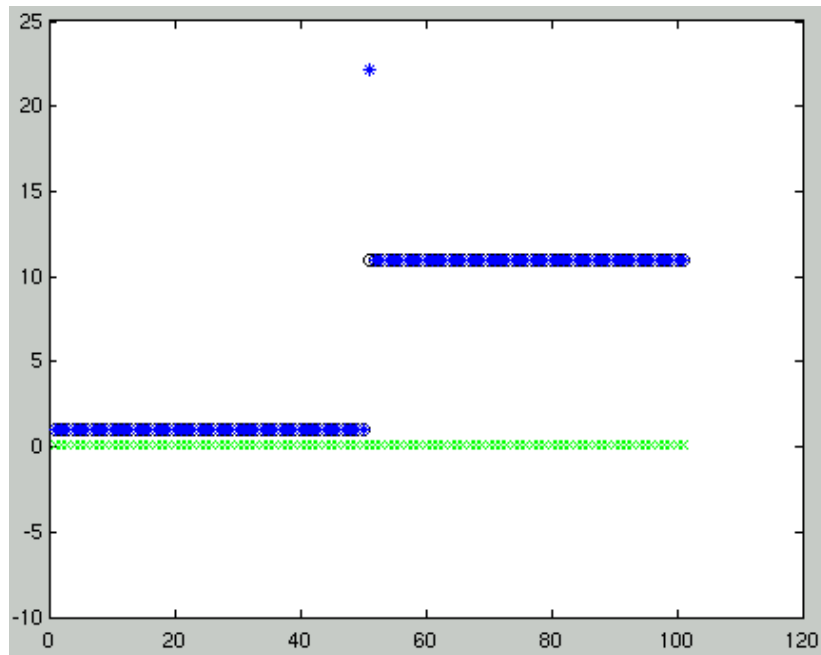


Gauss-Newton with projection
after 129 iterations

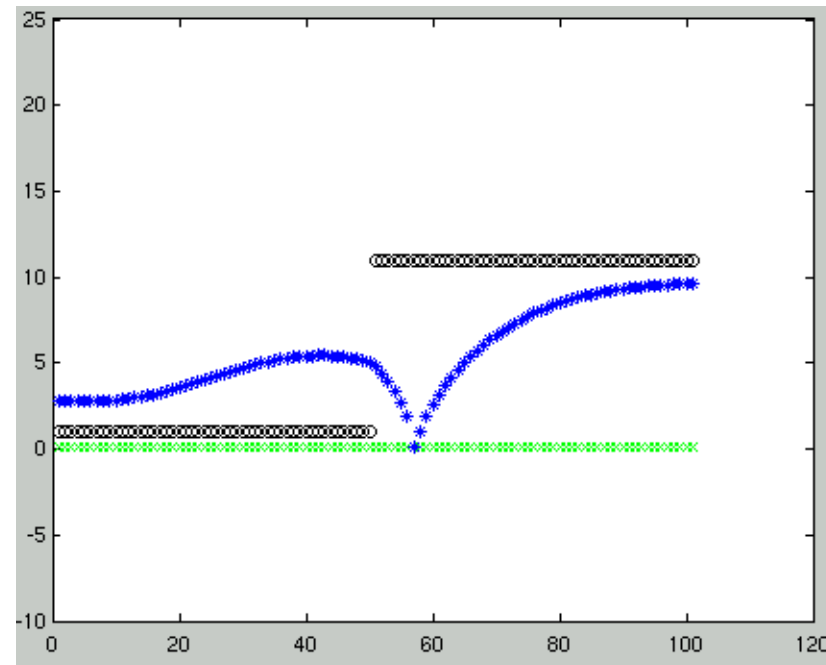


Numerical Tests (III)

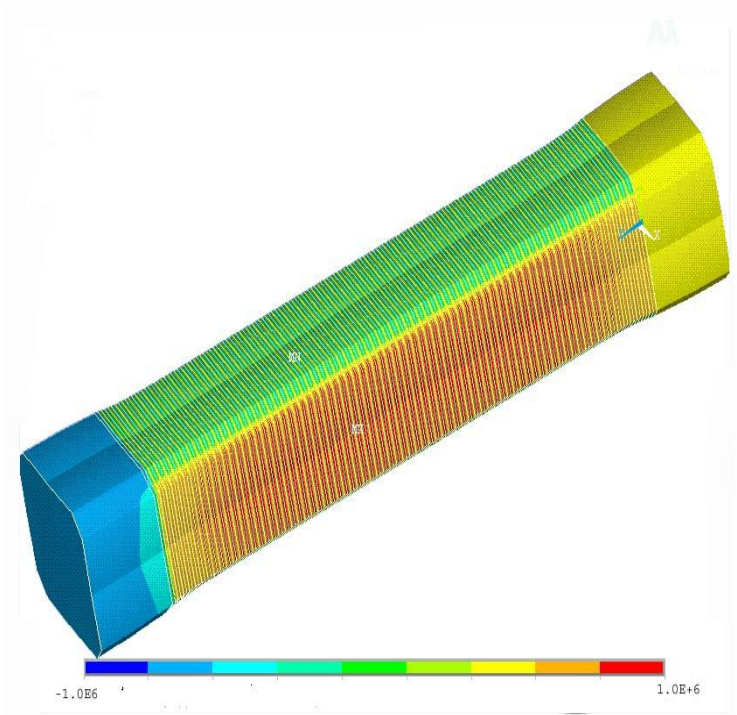
Levenberg-Marquardt with projection
after 105 iterations



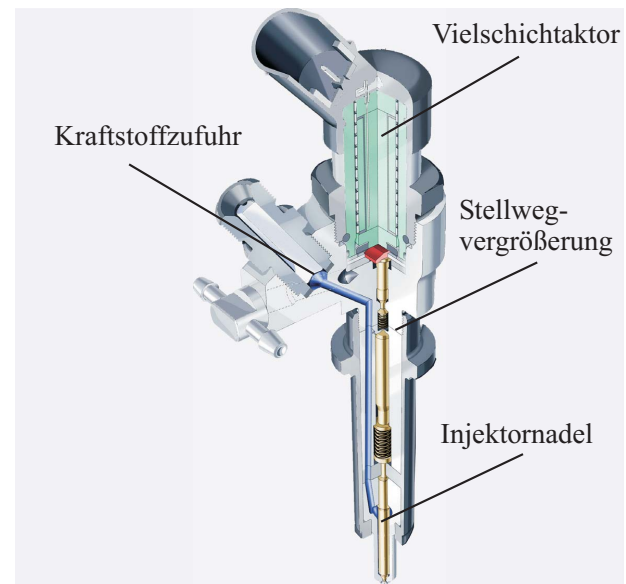
Landweber with projection
after 1000 iterations



Piezoelectric Stack Actuator



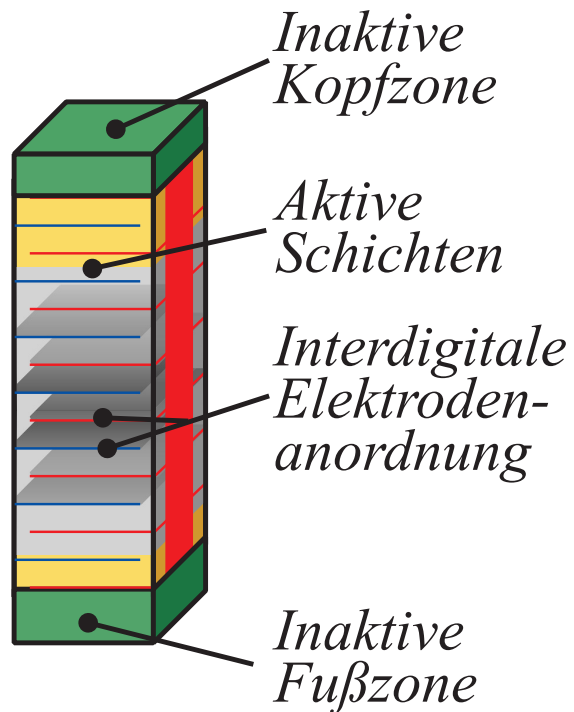
Injection valve Common-Rail-Dieselmotor



300-400 layers, \sim 130.000 unknowns,
CPU: 244.43 s

Homogenization for Piezoelectric Stack Actuator

joint work with Joachim Schöberl and Tom Lahmer



idea: compute solution as superposition of
global part + details on cells

challenge: *finite* periodic structure

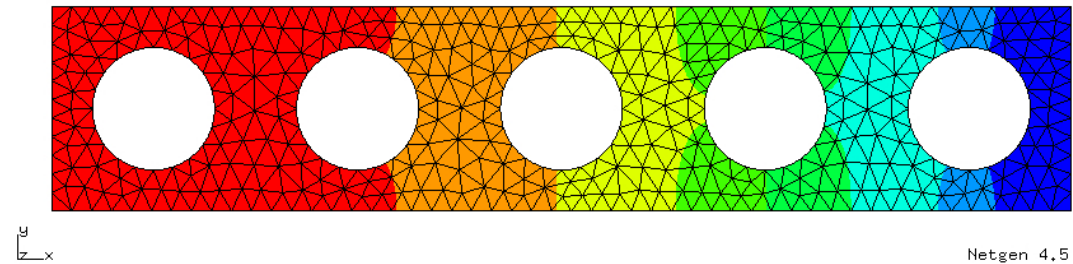
Homogenization

model problem

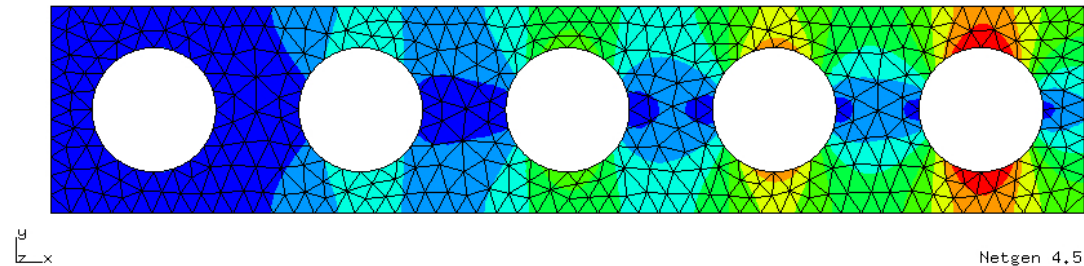
$$\begin{aligned}\Delta u &= f \text{ in } \Omega \\ u &= 0 \text{ on } \Gamma_D \\ \frac{\partial u}{\partial n} &= 0 \text{ on } \Gamma_N\end{aligned}$$

on p -periodic finite domain Ω .

solution:

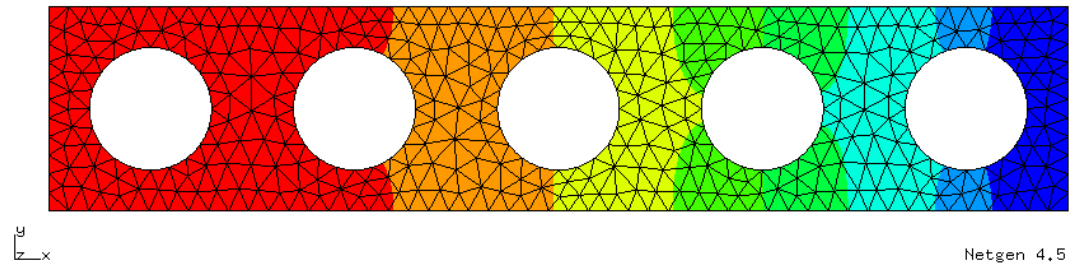


flux:



Homogenization Idea

$$\begin{aligned}\Delta u &= f \text{ in } \Omega \\ u &= 0 \text{ on } \Gamma_D \\ \frac{\partial u}{\partial n} &= 0 \text{ on } \Gamma_N\end{aligned}$$



Ansatz

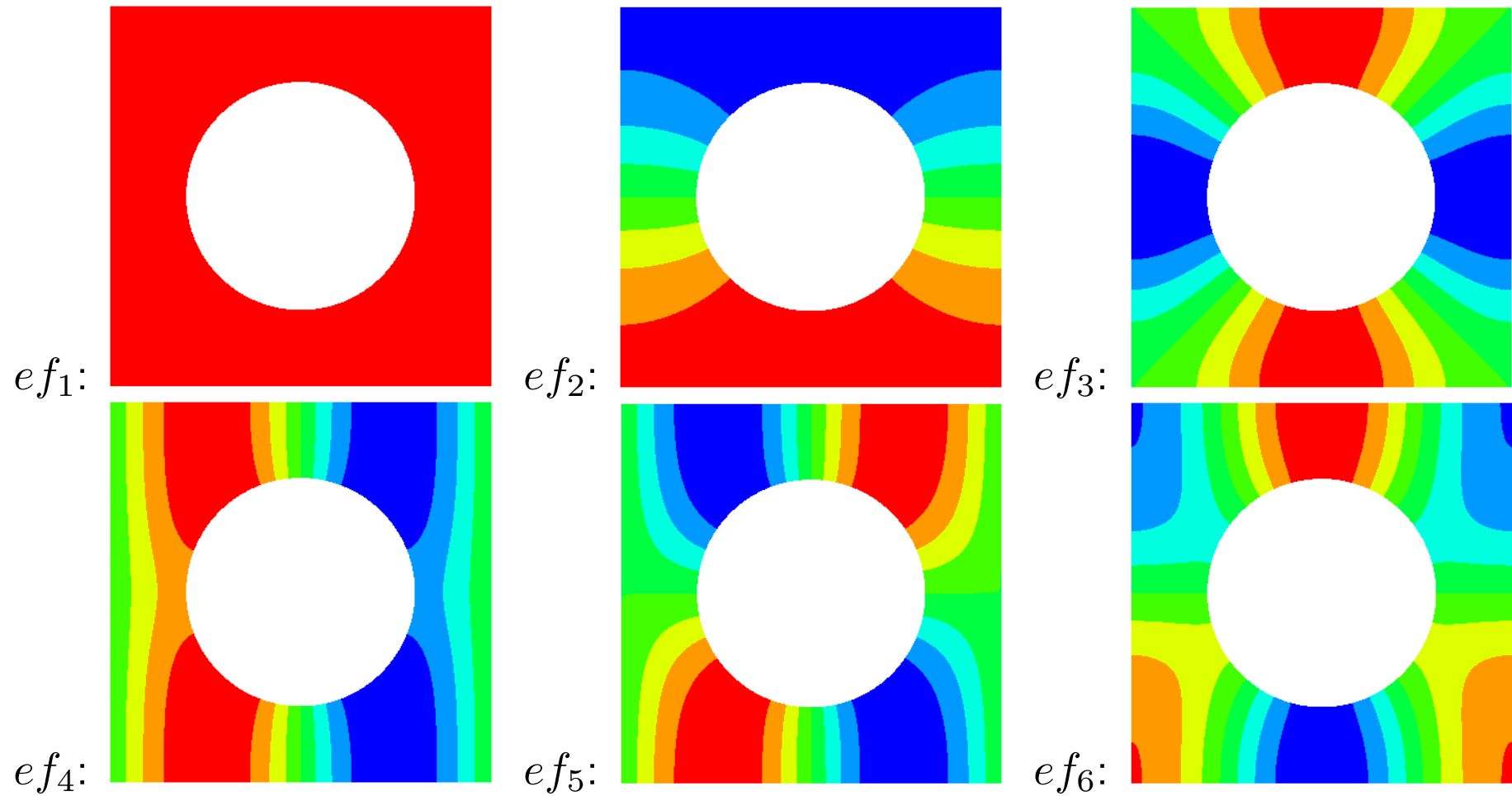
$$u(x, y) = \sum_{i=1}^N \underbrace{u_i(x, y)}_{\text{global}} \underbrace{\psi_i\left(\frac{x}{p}, y\right)}_{\text{periodic}}$$

cf. [Babuška&Morgan, Schwab&Matache&Rüegg] “gFEM”, “micro shape functions”

unit cell: we choose ψ_i as eigenfunctions with periodic Dirichlet BC

global: FEM with one element per cell (or per n cells) for easy assembling.

Eigenfunktionen on the Unit Cell



Numerical Tests

reference value: $f \cdot u = 38.190489$

$N_{ef} \setminus ord_{glob}$	1	2	3	4
1	29.58	29.77	34.09	34.78
2	29.6	29.77	35.32	35.58
3	29.7	30.33	36.31	37.29
4	32.5	35.11	37.69	38.18
5	34.5	37.54	37.88	38.19
6	34.5	37.54	37.96	38.20

$$DOFs = DOFs(ord_{glob}) * N_{ev}$$

ord_{glob}	1	2	3	4
$DOFs(ord_{glob})$	12	33	64	105

Discussions and New Contacts

- Samuel Armstutz: Topological asymptotics for piezoelectricity
- Marc Kamlah: Hysteresis modelling in piezoelectricity
- Volker Schulz: Optimum experiment design for piezoelectric material parameter id.
- Gen Nakamura: Identifiability of nonlinear coefficient curves
- Herbert Egger, Boris Vexler: Adaptivity in parameter identification

Further Activities

- Miniworkshop *Direct and Inverse Problems in Piezoelectricity*
 - 11 talks
 - 20-25 participants
- lecture *Parameter Identification in PDEs*
 - 14 lectures
 - 10-30 participants