Algebraic multilevel preconditioning using local Schur complements

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We consider an algebraic multilevel preconditioning technique for SPD matrices arising from finite element discretization of elliptic PDEs. In particular, we address the case of non-M matrices. The main emphasis is on building robust components of a multilevel method by exploiting the additional information of individual element matrices. This approach is distinguished by avoiding the hierarchical basis representation.

The left upper block of the considered multiplicative two-level preconditioner is approximated using incomplete LU (ILU) factorization techniques. A favourable algorithm, which improves the usual modified ILU method (satisfying a row-sum criterion), can be derived based on element agglomeration. The coarse-grid element matrices are simply Schur complements computed from local neighborhood matrices, i.e., small collections of element matrices. Assembling these local Schur complements results in a global Schur complement approximation that can be analyzed by regarding (local) macro elements.

These components, when combined in the framework of an algebraic multilevel iteration, yield a robust and efficient linear solver. The presented numerical experiments include also the Lamé differential equation for the displacements in the two-dimensional plane-stress elasticity problem.