Abstract.
Growth terms are increasingly utilized in the continuum mechanics literature, particularly in connection with growing biological materials, such as tumor growth. Growth terms are also of importance in various other fields, e.g. in the equations describing chemically reacting mixtures. In the present lecture, we point out some general consequences of introducing growth terms into the balance equations of continuum mechanics. We use the Eulerian description of continuum mechanics, in a framework that also includes polar (Cosserat-type) and micromorphic media. In the first part of the lecture, we derive relations that must necessarily hold between growth terms that appear in the fundamental equations of balance of continuum mechanics, namely balance of mass, linear momentum, angular momentum, total energy and entropy, and the growth terms that appear in the equations of balance that can be derived from the latter, namely balance of moment of momentum, spin, translatory kinetic energy, rotatory kinetic energy, internal energy and Helmholtz free energy. As a by-product and for the non-polar case, we suggest a continuum model of growth that is motivated by the Stefan law of diffusion and by formulations dating back to Buquoy, Seeeliger and Mescherskii for rockets. We show that some of the recent biological growth models from the literature are included in our formulation as special cases, and we are thus able to clarify an apparent contradiction in the literature. In the last part of the lecture, we extend the above consistency relations for growth terms appearing in the equations of balance with respect to consistency relations for surface growth terms appearing in the jump relations at a singular surface, on which certain entities take on different values when approaching from the two sides of that surface. For that sake, we first present an extension of the classical Kotchine jump relations, and we afterwards derive consistency relations between surface growth terms appearing in the fundamental jump relations (for mass, linear momentum, angular momentum energy and entropy) and those appearing in the jump relations for moment of momentum, spin, translatory kinetic energy, rotatory kinetic energy, internal energy and Helmholtz free energy.

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