Upscaling : A review by C.L. Farmer, 2002

Calculating permeability : A review by Ph Renard and G. de Marsily, 1996

———— x ———— x ———— x ———— x ————

Flow models : can be divided into two groups.

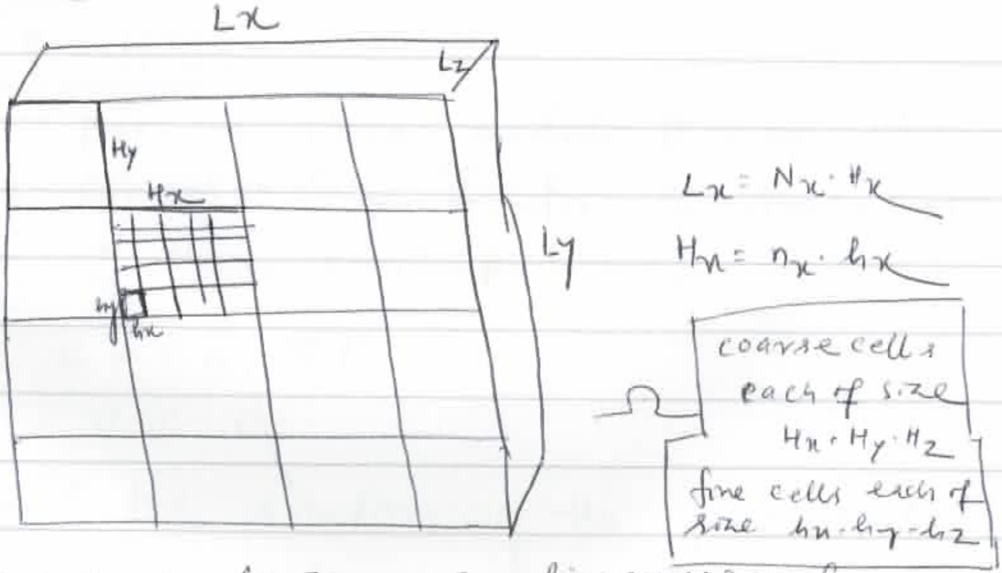
1- Those for which the parameters are obtained by calibration on an observed pressure record

2- Those for which there are no such record

upscaling

Objectives : - Define the upscaling problem.
- published techniques for calculating the permeability.

Model problem:



Let Ω be a finite 3-dimensional region with boundary $\partial\Omega$.

Let u be a flow vector with components (u, v, w) and p be the pressure field in an incompressible fluid in a porous medium.

Then two equations that describe the flow are:

1- Darcy's law: $u = -K \cdot \nabla P$

2- The mass balance equation: ——— (1)

$$\nabla \cdot u = 0$$

where K is the permeability tensor.

equivalent permeability (K_{eq})

A constant permeability tensor taken to represent a heterogeneous medium.

Effective permeability (K_{ef}):

A term used for a medium that is statistically homogeneous on a large scale.

Upscaling problem:

For the above model problem (1), to find a corresponding $\tilde{K}$ for K such that the solution of the coarse problem

$$\tilde{u} = -\tilde{K} \cdot \nabla \tilde{P}$$

$$\nabla \cdot \tilde{u} = 0$$

is near the solution of the fine scale problem

Methods:

1- Heuristic Methods
2- Deterministic Method
3- Stochastic Method

Integqualities for the equivalent permeability:

~~Wiener bounds~~

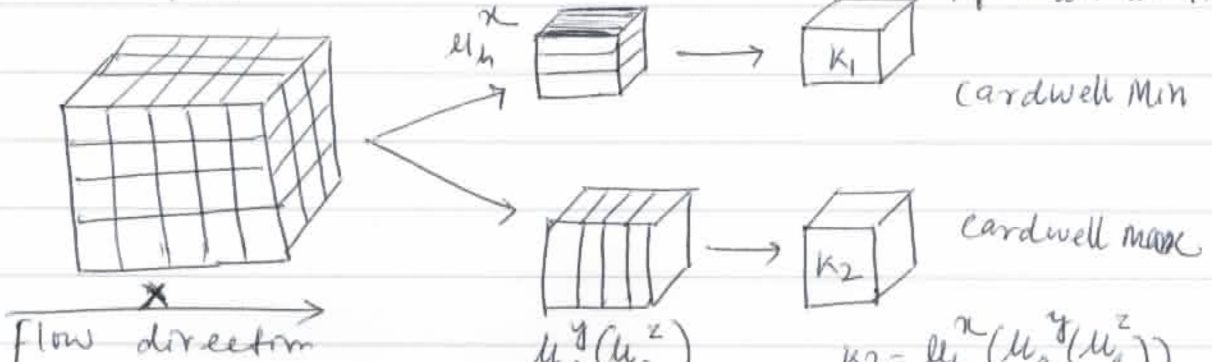
[O wiener, 1912, cardwell & parsons, 1945, G. Matheron, 1967, G. Dagan, 1989]

1- Wiener bounds (fundamental inequality)

$$\mu_h \le K_{ef} \le \mu_a$$

where μ_h = harmonic mean
and μ_a = arithmatic mean

$K_1 = \mu_a^z(\mu_a^y(\mu_h^x))$



$K_2 = \mu_h^x(\mu_a^y(\mu_a^z))$

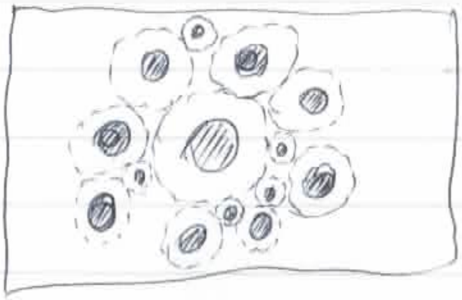
2- Hashin and Shtrikman bounds: [Hashin & Shtrikman, 1963]
(used for isotropic binary media)

$$\mu_a - \frac{f_1 f_0 (K_1 - K_0)^2}{(D - f_0) K_0 + f_0 K_1} \le K_{ef} \le \mu_a - \frac{f_1 f_0 (K_1 - K_0)^2}{(D - f_1) K_1 + f_1 K_0}$$

Where D = space dimension.

f_0 and f_1 are the fractions of the permeability phases K_0 and K_1.

$K_1 > K_0$ and $\mu_a = f_0 K_0 + f_1 K_1$



Each composite sphere is defined by an isotropic ~~spoter~~ sphere with a constant permeability K_{in} and an isotropic concentric sell with a K_{out} permeability.

Maximum permeability:

assume $K_{in} = K_0$, $K_{out} = K_1$

Minimum permeability:

assume $K_{in} = K_1$, $K_{out} = K_0$

3- Cardwell and Parsons bonds:

$$K_1 = \mu_h^x(\mu_a^z(\mu_a^y)) \leq K_{eq}^{xx} \leq K_2 = \mu_a^z(\mu_a^y(\mu_h^x))$$

4- Matheron bonds:

(In case of an isotropic two-dimensional random mosaic with two phases)

$f_0 \geq 0.5 \Rightarrow K_{ef} \geq K_{ac}$

$f_0 \leq 0.5 \Rightarrow K_{ef} \leq K_{ac}$

with

$$K_{ae} = \frac{1}{2}\left[(f_1 - f_0)(k_1 - k_0) + \sqrt{(f_1-f_0)^2(k_1-k_0)^2 + 4k_0k_1}\right]$$

Note: This relation is valid if the mosaic is invariant by a $90°$.

$$f_0 \geq 0.5 \Rightarrow k_{ef} \leq k_m$$

$$= \frac{f_1 k_0 k_1 + f_0 \mu_a \sqrt{k_0(2\mu_a - k_0)}}{f_1 m^* + f_0 \sqrt{k_0(2\mu_a - k_0)}}$$

$$f_0 \leq 0.5 \Rightarrow k_{ef} \geq k_m$$

$$= k_0 k_1 \frac{f_0 \mu_a + f_1 \sqrt{k_0(2m^* - k_0)}}{f_0 k_0 k_1 + f_1 m^* \sqrt{k_0(2m^* - k_0)}}$$

with $m^* = f_1 k_0 + f_0 k_1$

$\mu_a = f_1 k_1 + f_0 k_0$

$\sigma^2 = f_1 f_0 (k_1 - k_0)^2$

Heuristic Methods

1- Sampling:

- The first technique is simply not to change scales.
- A block is given the permeability measured at its center.

2- Averaging means:

- Take a value between two theoretical bounds.

fundamental bounds:

$$K_{ef} = \mu_a^{\alpha} \mu_h^{1-\alpha} \quad \text{with } \alpha \in [0,1] \quad \text{(G. Matheron, 1967)}$$

If the medium is statistically homogeneous and isotropic,

$$\alpha = \frac{D-1}{D}$$

In case of an anisotropic, but statistically homogeneous medium,

$$K_{ef}^{ii} = (\mu_a^{\alpha_i})(\mu_h^{1-\alpha_i}) \quad \text{(R. Ababou, 1995)}$$

$$\alpha_i = \frac{1}{D}\left(D - l_h/l_i\right)$$

where l_i = the correlation length in the relevant direction

l_h = the harmonic ~~length~~ mean of the correlation lengths in the principal directions of anisotropy

cardwell and parsons bounds:

[R. Knud-Romeu, 1994, D. Guerillot, 1990]

— Take the geometric mean of two cardwell and parsons bonds.

P. Lemouzy, 1991. generalizes this idea, for three dimensional media: block permeability tensor K_b is given by:

$$K_b^{xx} = \sqrt[6]{K_1^2 K_2^2 K_3 K_4}$$

with

$$K_1 = \mu_h^x(\mu_a^y(\mu_a^z)) = \mu_h^x(\mu_a^z(\mu_a^y))$$

$$K_2 = \mu_a^y(\mu_a^z(\mu_h^x)) = \mu_a^z(\mu_a^y(\mu_h^x))$$

$$K_3 = \mu_a^y(\mu_h^x(\mu_a^z))$$

$$K_4 = \mu_a^z(\mu_h^x(\mu_a^y))$$

R. Knud-Romeu 1994: ~~consts~~ introduces exponents that control the influence of anisotropy.

$$K_b^{xx} = K_1^{(\theta_{y2}\theta_{z3} + \theta_{z2}\theta_{y3})} K_2^{(1-\theta_{y3}-\theta_{z3})} K_3^{(1-\theta_{y2})\theta_{z3}} K_4^{(1-\theta_{z2})\theta_{y3}}$$

with

$$\theta_{y2} = \frac{\arctan\sqrt{a_y}}{\pi/2} \qquad \theta_{z2} = \frac{\arctan\sqrt{a_z}}{\pi/2}$$

$$\theta_{y3} = \frac{\theta_{y2}(1-\theta_{z2})}{1-\theta_{y2}\theta_{z2}} \qquad \theta_{z3} = \frac{\theta_{z2}(1-\theta_{y2})}{1-\theta_{y2}\theta_{z2}}$$

$$a_y = \frac{K^{yy}}{K^{xx}}\left(\frac{dx}{dy}\right)^2 \qquad a_z = \frac{K^{zz}}{K^{xx}}\left(\frac{dx}{dz}\right)^2$$

where a_y & a_z are the anisotropy factors

3- Power average:

[Journel et al. - 1986]

→ K_{ef} be equal to a power average (or average of order p) with an exponent p in the interval -1 and $+1$.

$$\mu_p = \langle k^p \rangle^{1/p} = \left(\frac{1}{V} \int_V K(x)^p \, dV \right)^{1/p}$$

Where μ_p = mean of order p

$\langle \rangle$ = average operature

Note: $p = -1$ corresponds to the harmonic mean

$\lim_{p \to 0} \mu_p$ to the geometric mean

and $p = 1$ to the arithmetic mean

For statistically homogeneous and isotropic media B. Noetinger, 1994 obtains:

$$p = 1 - \frac{2}{D}$$

In the case of a log-normal medium, Ababou et al - 1989 obtains:

$$\mu_p = \mu_g \exp\left(\frac{p \sigma^2_{\ln k}}{2} \right)$$

where μ_g: geometric mean

$\sigma^2_{\ln k}$ = variance of the logarithmic k

for a binary medium:

$$K_{ef} = \left[f_0 K_0^p + f_1 K_1^p \right]^{1/p}$$

Deterministic Method:

– The permeability field $k(x,y,z)$ and the boundary conditions are assumed to be known.

— for more general cases, there are the theories

percolation
effective medium
Streamline
renormalization

can be used to make approximated calculation with varying precision.

Example: Local method: [Warren. J. E. 1961)

– consider each coarse grid ~~coarse~~ cell separately ~~and p.~~

– performs three independent flow experiments with no flow boundary conditions on four sides of the cell.

– take constant pressure conditions on two opposing faces, say

$P_b = 1$ on one face

$P_b = 0$ on the other

– If the total flux is computed through one of the faces with pressure B.C., then equivalent permeability ~~over~~ in x-direction on the grid cell will be:

$$\tilde{K}_x = \frac{Q\, H_x}{A\, \Delta P}$$

where Q = the total flux through the face of area $A = H_y \cdot H_z$

ΔP = pressure drop.

– rotating the B.C.s ~~and repeating the flow solutions~~ the permeability in other directions can be obtained.

In one dimension local methods are exact:

The flow equations in one-dimension reduced to

$$\frac{\partial u}{\partial x} = 0, \quad \frac{u}{K} = -\frac{\partial P}{\partial x}$$

Integrating w.r.t x over one coarse grid cell, the above first equation shows that u is constant.

and second equation gives the value:

$$u = \frac{\Delta P_{I+1/2}}{\int_{I-1/2}^{I+1/2} \frac{1}{K}}$$

where $\Delta P_{I+1/2} = P_I - P_{I+1}$

and the limits $I-\frac{1}{2}$ and $I+\frac{1}{2}$ indicate that x belongs to cell $w_{I+1/2}$

From the definition

$$\tilde{k} = \frac{u H_x}{\Delta P_{i+1/2}}$$

i.e. the effective permeability in one coarse cell is found to be the harmonic average $\tilde{k} = H_x / \int \frac{1}{k}$

The total flux through all of the coarse grid cells, deduced from the coarse grid equation, is

$$\tilde{u} = \Delta P \int_0^{L_x} \frac{1}{\tilde{k}}$$

Where ΔP is the total pressure drop.

This is the exact result because

$$\int_0^{L_x} \frac{1}{\tilde{k}} = \int_0^{1/2} \frac{1}{k} + \sum_i \int_{i-1/2}^{i+1/2} \frac{1}{k} + \int_{L_x - H_x/2}^{L_x} \frac{1}{k}$$

Non local or global methods:

- This case considers the flow over the entire domain at the scale of the fine-mesh grid with several sets of b.c. on the sides of the domain.

Stochastic Methods:

Refer to Sergey & Aretha (next day)