On the impact of correlation on option prices: a Malliavin Calculus approach

RESULTS FROM

E. Alòs (2006): A generalization of the Hull and White formula with applications to option pricing approximation. *Finance and Stochastics* **10** (3), 353-365.

E. Alòs, J. A. León and J. Vives (2007): On the short-time behaviour of the implied volatility for stochastic volatility models with jumps. *Finance and Stochastics* **11** (4), 571-589

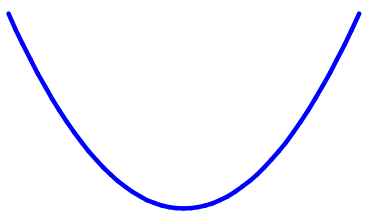
STOCHASTIC VOLATILITY MODELS

Stochastic volatility models allow us to describe the smiles and skews observed in real market data:

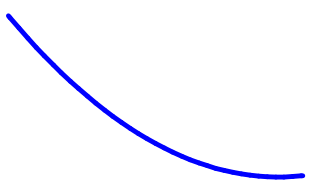
$$dX_t = \left(r - \frac{1}{2}\sigma_t^{\,2} \right) dt + \sigma_t \left(\rho dW_t^* + \sqrt{1-\rho^2}\, dB_t^* \right)$$

Log-price

Volatility (stochastic, adapted to the filtration generated by W)



Implied volatility smile $\rho = 0$



Implied volatility skew $\rho \neq 0$

SOME QUESTIONS AND MOTIVATION

How to quantify the impact of correlation on option prices?

What about the term structure?

We will develop a formula of the form

Option price
=option price in the uncorrelated case
(classical Hull and White formula)
+ correction due by correlation

This result will allow us to describe the impact of the correlation on the option prices. As an application, we can use it to construct option pricing approximation formulas, or to study the short-time behaviour of the implied volatility for stochastic volatility models with jumps.

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (I)

Here our pourpose is to present the basic concepts on Malliavin calculus that have been used up to now in financial applications. Basically, we will see how to:

Calculate the Malliavin derivative of a diffusion process

Use the duality relationship between the Malliavin derivative and the Skorohod integral to develop adequate change-of-variable formulas for anticipating processes

MAIN IDEA: THE FUTURE INTEGRATED VOLATILITY IS AN ANTICIPATING PROCESS

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (II)

<u>Malliavin derivative : definition</u>

$$\{W(h), h \in L^2([0,T])\}$$

Gaussian process

$$F = f(W(h_1), W(h_2) \ldots, W(h_n))$$

random variable in $L^2([0,T] \times \Omega)$

$$\boxed{D_t F = \sum \frac{\partial f}{\partial x_i}(W(h_1), W(h_2) \ldots, W(h_n)) h_i(t)}$$

Malliavin Derivative in $L^2([0,T] \times \Omega)$

(closable operator)

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (III)

Malliavin derivative: examples

$$\boxed{D_t^W W_s = 1_{[0,s]}(t)}$$

$$S_t = \exp\left\{\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\} \text{(Black-Scholes)}$$

$$D_r^W S_t = \sigma S_t 1_{[0,t]}(r)$$

$$Y_t = m + (Y_0 - m)e^{-\alpha t} + c\int_0^t e^{-\alpha(t-r)} dW_r$$

$$(Ornstein - Uhlenbeck)$$

$$D_r^W Y_t = ce^{-\alpha(t-r)} 1_{[0,t]}(r)$$

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (IV)

Skorohod integral: definition

It is the adjoint of the Malliavin derivative operator:

$$\boxed{E\left(\delta^{W}(u)F\right) = E\int_0^T \left(D_s^W F\right) u_s\, ds \text{ , for all } F \in S}$$

Example:

$$h \in L^2([0,T]) \Rightarrow \delta^W(h) = W(h)$$

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (V)

Skorohod integral: properties

The Skorohod integral of a process multiplied by a random variable

$$\int_0^T F u_s dW_s = F \int_0^T u_s dW_s + \int_0^T \left(D_s^W F \right) u_s ds$$

The Skorohod integral is an extension of the classical Itô integral

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (VI)

THE ANTICIPATING ITÔ'S FORMULA

$$X_t = X_0 + \int_0^t u_s dW_s + \int_0^t v_s ds$$

Non necessarily adapted

$$\boxed{\begin{aligned} F(X_t) &= F(X_0) + \int_0^t F'(X_s) u_s dW_s \\ &\quad + \int_0^t F'(X_s) v_s ds + \frac{1}{2}\int_0^t F''(X_s)(\nabla u)_s u_s ds, \end{aligned}}$$

where $(\nabla u)_s := u_s + 2\int_0^s D_s^W u_r dW_r + 2\int_0^s D_s^W v_r dr$

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (VI)

Proof (sketch)

For the sake of simplicity we assume $v \equiv 0$.

We proceed as in the proof of the classical Itô's formula

$$F(X_t) = F(X_0) + \sum F'(X_{t_i})\left(\int_{t_i}^{t_{i+1}} u_s dW_s\right)$$

$$+\frac{1}{2}\sum F''(\overline{X}_{t_i})\left(\int_{t_i}^{t_{i+1}} u_s dW_s\right)^2$$

$$\frac{1}{2}\int_0^t u_s^2 ds$$

SOME PRELIMINARIES ON STOCHASTIC CALCULUS FOR ANTICIPATING PROCESSES (VII)

$$\sum F'\left(X_{t_i}\right)\left(\int_{t_i}^{t_{i+1}} u_s dW_s\right)$$

$$= \sum \int_{t_i}^{t_{i+1}} F'\left(X_{t_i}\right) u_s dW_s - \sum \int_{t_i}^{t_{i+1}} D_s^W F'\left(X_{t_i}\right) u_s ds$$

$$\frac{1}{2}\int_0^t F'(X_s) u_s dW_s$$

$$\underbrace{\int_0^t F''(X_s)\left(\int_0^s D_s^W u_r dW_r\right) u_s ds}_{\int_0^t D_s^{W-}\left[F'\left(X_s\right)\right] u_s ds}$$

AN EXTENSION OF THE HULL AND WHITE FORMULA (I)

<u>Basic idea</u>

The Black-Scholes function final condition implies that

$$V_T = BS\left(T, X_T; \nu_T\right)$$

Black-Scholes pricing formula

Log-price

$$\left(v_t^2 = \frac{1}{T-t}\int_t^T \sigma_s^2 ds\right)$$

where

$$V_t = E^*\left[e^{-r(T-t)} H \middle| F_t\right]$$

option price

payoff

AN EXTENSION OF THE HULL AND WHITE FORMULA (II)

Then

$$\boxed{\begin{aligned} V_t &= e^{-r(T-t)} E^*\left(V_T \middle| F_t\right) \\ &= E^*\left[BS\left(T, X_T; \nu_T\right) \middle| F_t\right] \end{aligned}}$$

compare

The classical Hull and White term

$$\boxed{E^*\left(BS\left(t, X_t; \nu_t\right) \middle| F_t\right)}$$

is the option price in the uncorrelated case

AN EXTENSION OF THE HULL AND WHITE FORMULA (III)

Then we want to evaluate the difference

$$\boxed{BS\left(T, X_T; \nu_T\right) - BS\left(t, X_t; \nu_t\right)}$$

ν_t is an anticipating process

$\downarrow$

We need to construct an adequate anticipating Itô's formula

AN EXTENSION OF THE HULL AND WHITE FORMULA (IV)

Anticipating Itô's formula

$$F(t, X_t, Y_t) = F(0, X_0, Y_0) + \int_0^t \frac{\partial F}{\partial s}(s, X_s, Y_s)ds$$

$$+ \int_0^t \frac{\partial F}{\partial x}(s, X_s, Y_s)dX_s + \int_0^t \frac{\partial F}{\partial y}(s, X_s, Y_s)dY_s$$

$$+ \boxed{\int_0^t \frac{\partial^2 F}{\partial x \partial y}(s, X_s, Y_s)(D^- Y)_s u_s ds}$$

additional term

$$+ \int_0^t \frac{\partial^2 F}{\partial x^2}(s, X_s, Y_s)u_s^2 ds$$

$$Y_t = \int_t^T \theta_s ds \qquad (D^- Y)_s := \int_s^T D_s^W \theta_r dr$$

AN EXTENSION OF THE HULL AND WHITE FORMULA (V)

Main result:

the extension of the Hull and White formula

We apply the above Itô's anticipating formula
to the process

$$\boxed{e^{-rt} BS\left(t, X_t; \nu_t\right) = e^{-rt} BS\left(t, X_t; \sqrt{\frac{1}{T-t} Y_t}\right)}$$

and we obtain

AN EXTENSION OF THE HULL AND WHITE FORMULA (VI)

$$e^{-rT}V_T = e^{-rT}BS(T, X_T, \nu_T) = e^{-rt}BS(t, X_t, \nu_t)$$

$$+\int_t^T e^{-rs}\left[L_{BS}(\nu_s) + \frac{1}{2}\left(\sigma_s^2 - \nu_s^2\right)\left(\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial x}\right)\right]BS(s, X_s, \nu_s)ds$$

$$+\int_t^T e^{-rs}\frac{\partial BS}{\partial x}(s, X_s, \nu_s)\sigma_s\left(\rho dW_s^* + \sqrt{1-\rho^2}dZ_t^*\right)$$

$$+\frac{\rho}{2}\int_0^t e^{-rs}\frac{\partial^2 BS}{\partial x \partial \sigma}(s, X_s, \nu_s)\frac{1}{\nu_s(T-s)}\left(D^- Y\right)_s \sigma_s ds$$

$$-\frac{1}{2}\int_t^T e^{-rs}\frac{\partial BS}{\partial \sigma}(s, X_s, \nu_s)\frac{\left(\sigma_s^2 - \nu_s^2\right)}{\nu_s(T-s)}ds$$

Black-Scholes differential operator

Cancel

Zero expectation

AN EXTENSION OF THE HULL AND WHITE FORMULA (VII)

$$e^{-rT}E^*\left(V_T \middle| F_t\right) = e^{-rt}E^*\left(BS\left(t, X_t, \nu_t\right) \middle| F_t\right)$$

$$+\frac{\rho}{2}\int_0^t e^{-rs}\underbrace{\frac{\partial^2 BS}{\partial x \partial \sigma}\left(s, X_s, \nu_s\right)\frac{1}{\nu_s(T-s)}}_{\left(\frac{\partial^3}{\partial x^3}-\frac{\partial^2}{\partial x^2}\right)\left(s, X_s, \nu_s\right) =: H\left(s, X_s, \nu_s\right)}\underbrace{\left(D^- Y\right)_s \sigma_s}_{\Lambda_s := \left(\int_s^T D_s^{W^*}\sigma_r^2 dr\right)\sigma_s} ds$$

$$\boxed{\begin{aligned} V_t &= e^{-r(T-t)}E^*\left(V_T \middle| F_t\right) \\ &= E^*\left(BS\left(t, X_t, \nu_t\right) \middle| F_t\right) \\ &+ \frac{\rho}{2}E^*\left(\int_0^t e^{-r(s-t)}H\left(s, X_s, \nu_s\right)\Lambda_s ds \middle| F_t\right) \end{aligned}}$$

AN EXTENSION OF THE HULL AND WHITE FORMULA (VIII)

The above arguments do not requiere the volatility to be Markovian.

The main contribution of this formula is to describe the effect of the correlation as the term

$$\boxed{\frac{\rho}{2} E^{*}\left(\int_0^t e^{-r(s-t)} H(s, X_s, \nu_s) \Lambda_s ds \middle| F_t \right)}$$

APPLICATIONS TO OPTION PRICING APPROXIMATION (I)

Consider the approximation

$$V_{aprox} = BS\left(t, X_t, \nu_t^*\right) + \frac{\rho}{2} H\left(t, X_t, \nu_t^*\right) E^*\left(\int_t^T \Lambda_s ds \middle| F_t \right),$$

$$\nu_t^* = \sqrt{\frac{1}{T-t} \int_t^T E^*\left(\sigma_s^2 \middle| F_t\right) ds}$$

APPLICATIONS TO OPTION PRICING APPROXIMATION (II)

Using Malliavin calculus again we can see that, in the case

$$\sigma_s = f(Y_s), \text{ with}$$

$$dY_t = \alpha(m - Y_t)dt + \lambda\sqrt{\alpha}dW_t$$

and f regular enough

$$\Downarrow$$

$$\boxed{\left|V_t - V_{aprox}\right| \leq C\frac{\lambda^2}{\alpha}\left(1 + \left|\ln\alpha\right|\right)}$$

A similar result was proven by Alòs and Ewald (2008) for the Heston volatility model

APPLICATIONS TO OPTION PRICING APPROXIMATION (III)

Application:

the Stein and Stein model with correlation

$$(\sigma_t = Y_t)$$

$$c = \lambda\sqrt{\alpha},$$

$$\boxed{(v_t^*)^2 = \frac{1}{T-t}\int_t^T \left(M_t^2(s) + c^2 F(s-t)\right)ds}$$

$$M_t(s) = m + (\sigma_t - m)e^{-\alpha(s-t)},\ F(u) = \int_0^u e^{-2\alpha\theta}d\theta$$

$$\boxed{\begin{aligned} &E^*\left(\int_t^T \left(\int_s^T Y_r e^{-\alpha(r-s)}dr\right)Y_s ds \middle| F_s\right) \\ &= c\int_t^T M_t(s)\left(\int_s^T e^{-\alpha(r-s)}M_t(r)dr\right)ds + c^2\int_t^T F(T-s)F(s)ds \end{aligned}}$$

APPLICATIONS TO OPTION PRICING APPROXIMATION (IV)

Numerical results

$$(T-t=0.5, X_t=\ln 100, \alpha=4, m=0.2, \lambda=0.05, r=0.0953, \sigma_t=0.2)$$

ρ	K	Exact price	Approximation	Pricing error (Percent)
-0.5	90	15.292	15.298	0.039
-0.5	95	11.503	11.521	0.156
-0.5	100	8.243	8.270	0.328
-0.5	105	5.595	5.627	0.572
-0.5	110	3.582	3.613	0.865
0.5	90	15.003	15.006	0.020
0.5	95	11.243	11.261	0.160
0.5	100	8.106	8.135	0.358
0.5	105	5.640	5.672	0.567
0.5	110	3.803	3.832	0.763

APPLICATIONS TO THE STUDY OF LONG-MEMORY VOLATILITY MODELS (I)

Example: long-memory volatilities

Assume that (see for example Comte, Coutin and Renault (2003)),

$$\sigma_t^2 = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} \tilde{\sigma}_s^2 ds,$$

$$\text{where } \tilde{\sigma}_s = f(Y_s)$$

$$\Downarrow$$

$$\int_s^T \sigma_r^2 dr = \frac{1}{\Gamma(\beta+1)} \int_s^T (T-r)^{\beta} \tilde{\sigma}_r^2 du,$$

$$+ \int_s^T \left(\int_0^s \frac{(r-u)^{\beta-1}}{\Gamma(\beta)} \tilde{\sigma}_u^2 du \right) dr$$

APPLICATIONS TO THE STUDY OF LONG-MEMORY VOLATILITY MODELS (II)

$$\int_s^T D_s^{W*}\sigma_r^2 dr = \frac{1}{\Gamma(\beta+1)}\int_s^T (T-r)^\beta D_s^{W*}\tilde{\sigma}_r^2 dr$$

$$E^*\left(\int_t^T \Lambda_s ds \middle| F_t\right)$$

$$= E^*\left(\frac{\lambda\sqrt{\alpha}\rho}{\Gamma(\beta+1)}\int_t^T\left(\int_s^T (T-r)^\beta D_s^{W*}\tilde{\sigma}_r^2 dr\right)\tilde{\sigma}_s ds \middle| F_t\right)$$

$$= \frac{2\lambda\sqrt{\alpha}\rho}{\Gamma(\beta+1)}$$

$$\times E^*\left(\int_t^T\left(\int_s^T (T-r)^\beta e^{-\alpha(r-s)} f(Y_r) f'(Y_r) dr\right) f(Y_s) ds \middle| F_t\right)$$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY FOR JUMP-DIFFUSION MODELS WITH STOCHASTIC VOLATILITY (I)

In Alòs, León and Vives (2007) we considered the following model for the log-price of a stock under a risk-neutral probability Q:

$$\boxed{\begin{aligned} X_t &= x + (r - \lambda k)t - \frac{1}{2}\int_0^t \sigma_s^2 ds \\ &\quad + \int_0^t \sigma_s \left(\rho dW_s + \sqrt{1-\rho^2}\, dB_s\right) + Z_t,\ t \in [0,T] \end{aligned}}$$

Adapted to the filtration generated by W

independent

Compound Poisson with intensity λ and Lévy measure $\nu = \lambda f(y)$, with $k = \frac{1}{\lambda}\int \left(e^y - 1\right)\nu(dy) < \infty$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (II)

Similar arguments as in the previous paper give us the following extension of the Hull and White formula:

$$V_t = E\left(BS(t, X_t, \nu_t) \middle| F_t\right) \qquad \text{Hull and White}$$

$$+ \frac{\rho}{2} E\left(\int_0^t e^{-r(s-t)} \partial_x G(s, X_s, \nu_s) \Lambda_s ds \middle| F_t\right) \qquad \text{Correlation}$$

$$+ E\left(\int_t^T \int_R e^{-r(s-t)} \left(BS(s, X_s + y, \nu_s) - BS(s, X_s, \nu_s)\right) \nu(dy) ds \middle| F_t\right)$$

$$- \lambda k E\left(\int_t^T e^{-r(s-t)} \partial_x BS(s, X_s, \nu_s) ds \middle| F_t\right) \qquad \text{Jumps}$$

$$\nu(t) = \sqrt{\frac{Y_t}{T-t}}; \quad G(t, x, \sigma) = \left(\partial_{xx}^2 - \partial_x\right) BS(t, x, \sigma)$$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (III)

After some algebra, we can prove from this expression that:

If $E\left[\left(D_s\sigma_r\right)^2 \middle| F_t\right] \leq C(r-s)^{2\delta}, \delta \geq 0$

and

$\lim_{r\downarrow t}\left[D_t\sigma_r\right] = D_t^+\sigma_t$

$$\boxed{\frac{\partial I_t}{\partial X_t}\left(x_t^*\right) \to -\frac{\rho}{\sigma_t} D_t^+\sigma_t - \frac{\lambda k}{\sigma_t}}$$

If $E\left[\left(D_s\sigma_r\right)^2 \middle| F_t\right] \leq C(r-s)^{2\delta}, \delta < 0$

and

$$\frac{1}{(T-t)^{2+\delta}} \int_t^T \int_s^T E\left(D_s\sigma_r \middle| F_t\right) dr ds \to L_t^{\delta,+}\sigma_t$$

$$\boxed{\lim (T-t)^{-\delta} \frac{\partial I_t\left(x_t^*\right)}{\partial X_t} = -\frac{\rho}{\sigma_t} L_t^{\delta,+}\sigma_t}$$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (IV)

Example 1: classical jump-diffusion models

Assume that the volatility process can be written as

$$\sigma_r = f(Y_r),$$

$$dY_r = a(r, Y_r)dr + b(r, Y_r)dW_r$$

$$\Downarrow$$

$$D_s Y_r = \int_s^r \frac{\partial a}{\partial x}(u, Y_u) D_s Y_u du + b(s, Y_s)$$

$$+ \int_s^r \frac{\partial b}{\partial x}(u, Y_u) D_s Y_u dW_u$$

$$\Downarrow$$

$$D_t^+ \sigma_t = f'(Y_t) b(t, Y_t)$$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (V)

$$\lim_{T\to t}\frac{\partial I_t}{\partial x}\left(x_t^*\right)=-\frac{1}{\sigma_t}\left(\lambda k+\rho f'\left(Y_t\right)b(t,Y_t)\right)$$

If Y is an Ornstein-Uhlenbeck process of the form

$$Y_r=m+\left(Y_t-m\right)e^{-\alpha(r-t)}+c\int_t^r\sqrt{2\alpha}e^{-\alpha(r-s)}dW_r$$

$$\lim_{T\to t}\frac{\partial I_t}{\partial x}\left(x_t^*\right)=-\frac{1}{\sigma_t}\left(\lambda k+\rho c\sqrt{2\alpha}\,f'\left(Y_t\right)\right)$$

(this agrees with the results in Medvedev and Scaillet (2004))

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (VI)

Example 2: fractional stochastic volatility models with H>1/2

Assume that the volatility process can be written as

$$\sigma_r = f(Y_r); \quad Y_r = m + (Y_t - m)e^{-\alpha(r-t)} + c\underbrace{\sqrt{2\alpha}\int_t^r e^{-\alpha(r-s)} dW_r^H}_{\left(H-\frac{1}{2}\right)\int_t^r \left(\int_s^r e^{-\alpha(r-u)}(u-s)^{H-\frac{1}{3}} du\right) dW_s}$$

$$\Downarrow$$

$$D_t^+ \sigma_t = 0$$

$$\Rightarrow \quad \boxed{\lim_{T\to t} \frac{\partial I_t}{\partial x}\left(x_t^*\right) = -\frac{\lambda k}{\sigma_t}}$$

That is, the at-the-money short-dated skew slope is not affected by the correlation in this case

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (VII)

Example 3: fractional stochastic volatility models with H<1/2

Assume that the volatility process can be written as

$$\sigma_r = f(Y_r); \quad Y_r = m + (Y_t - m)e^{-\alpha(r-t)} + c\sqrt{2\alpha}\int_t^r e^{-\alpha(r-s)} dW_r^H$$

$$\left(\frac{1}{2} - H\right)\int_t^r \left(\int_s^r \left(e^{-\alpha(r-u)} - e^{-\alpha(r-s)}\right)(u-s)^{H-\frac{1}{3}} du\right) dW_s$$

$$+\int_t^r e^{-\alpha(r-s)}(r-s)^{H-\frac{1}{2}} dW_s$$

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (VIII)

$$E\left(\frac{1}{(T-t)^{2+H-\frac{1}{2}}}\int_t^T\int_s^T D_s^W\sigma_r\,drds - c\sqrt{2\alpha}\,f'(Y_t)\,\middle|\,F_t\right)$$

$\rightarrow 0$ as $T \rightarrow t$.

$$\boxed{\lim_{T\rightarrow t}(T-t)^{\frac{1}{2}-H}\frac{\partial I_t}{\partial X_t}\left(x_t^*\right) = -c\rho\sqrt{2\alpha}\,f'(Y_t)}$$

That is, the introduction of fractional components with Hurst index H<1/2 in the definition of the volatility process allows us to reproduce a skew slope of order

$$O(T-t)^{\delta},\ \delta > -\frac{1}{2}$$

More similar to the ones observed in empirical data (see Lee (2004))

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (IX)

<u>Example 4: Time-varying coefficients</u>

(Fouque, Papanicolaou, Sircar and Solna (2004))

Assume that the volatility process can be written as

$$\sigma_r = f(Y_r)$$

$$Y_r = m + (Y_t - m)e^{-\int_t^r \alpha(s)ds} + c\int_t^r \sqrt{2\alpha(s)}e^{-\alpha(r-s)}dW_r \ ,$$

$$\sqrt{\alpha(s)} = (T - s)^{-\frac{1}{2}+\varepsilon}, \ \varepsilon > 0$$

Next maturity date

APPLICATIONS TO THE STUDY OF THE SHORT-TIME BEHAVIOUR OF THE IMPLIED VOLATILITY (X)

Then

$$\frac{1}{(T-t)^{2+\left(\frac{1}{2}-\varepsilon\right)}}\int_t^T\int_s^T E\left(D_s\sigma_r \middle| F_t\right)drds$$

$$+\rho c\left(\frac{1}{-1/2+\varepsilon}\right)\left(\frac{1}{1/2+\varepsilon}\right)\frac{f'(Y_t)}{2}$$

tends to zero as $T \to t$

In this case, the short-date skew slope of the implied volatility is of the order

$$O(T-t)^{-\frac{1}{2}+\varepsilon}$$

BIBLIOGRAPHY

E. Alòs (2006): A generalization of the Hull and White formula with applications to option pricing approximation. *Finance and Stochastics* **10** (3), 353-365.

E. Alòs, J. A. León and J. Vives (2007): On the short-time behaviour of the implied volatility for stochastic volatility models with jumps. *Finance and Stochastics* **11** (4), 571-589

E. Alòs and C. O. Ewald (2008): Malliavin Differentiability of the Heston Volatility and Applications to Option Pricing. *Advances in Applied Probability* **40** (1), 144-162.

F. Comte, L. Coutin and E. Renault (2003): Affine fractional stochastic volatility models with application to option pricing. Preprint.

J. P. Fouque, G. Papanicolau, K. R. Sircar and K. Solna (2004): Maturity Cycles in Implied Volatilities. *Finance and Stochastics* **8** (4), 451-477.

R. Lee (2004): Implied volatility: statics, dynamics and probabilistic interpretation. *Recent advances in applied probability*. Springer.

A. Medvedev and O. Scaillet (2004): A simple calibration procedure of stochastic volatility models with jumps by short term asymptotics. Discussion paper HEC, Gèneve and FAME, Université de Gèneve.