Analyzing market microstructure

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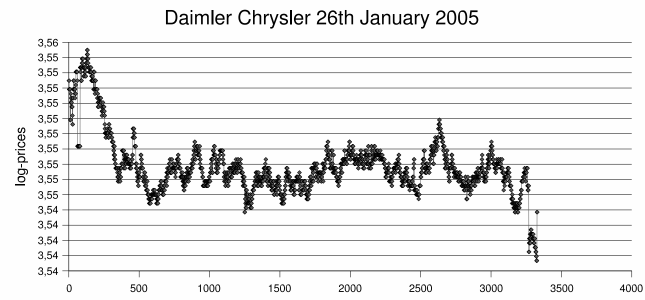
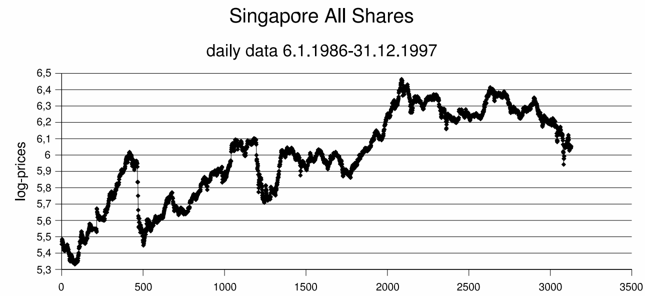
- Comparison of models based on
  - -Brownian motion
  - -Brownian motion with iid noise
  - -fractional Brownian motion
- Identification of jump components

Motivation



Daimler Chrysler 26th January 2005
log-prices
3,56
3,55
3,55
3,55
3,55
3,55
3,55
3,55
3,55
3,55
3,55
3,54
3,54
3,54
3,54
3,54
0
500
1000
1500
2000
2500
3000
3500
4000



Singapore All Shares
daily data 6.1.1986-31.12.1997
log-prices
6,5
6,4
6,3
6,2
6,1
6
5,9
5,8
5,7
5,6
5,5
5,4
5,3
0
500
1000
1500
2000
2500
3000
3500

Motivation

discrete data $X_{t_{n,0}}, \cdots, X_{t_{n,n}}$
$t_{n,n} = t =$fixed, $\Delta_{n,i} \to 0$ as $n \to \infty$
Assume stochastic volatility model

$$\begin{aligned} X_t &= Y_t + \int_0^t \sigma_s dB_s + \delta Z_t \\ X_t &= Y_t + \int_0^t \sigma_s dL_s + \delta Z_t \\ X_t &= Y_t + \int_0^t \sigma_s dB_s^H + \delta Z_t . \end{aligned}$$

Aim:

Estimate $\int_0^t \sigma_s^2 ds$ **or** $\int_0^t \sigma_s^p ds$**.** Does this work for all sampling frequencies?

How can we infer $\int_0^t \sigma_s^2 ds$

First we consider **Brownian motion based models**.
Use the concept of **quadratic variation**, i.e. realized volatility

$$\sum_i |Y_{t_i} - Y_{t_{i-1}} + \int_{t_{i-1}}^{t_i} \sigma_s dB_s|^2 \xrightarrow{p} \int_0^t \sigma_s^2 ds$$

Advantages:

- almost model free, only need some Brownian motion based model
- very simple to compute
- distributional theory is known and Gaussian

Empirical studies versus theoretical results

Statistical principle:

Use all available data.

Problem:
For tick-by-tick data realized volatility increases.
Possible Explanation:

Market microstructure or **market friction**

i.e. effects due to bid-ask bounces, discreteness of prices, liquidity problems, asymmetric information,...

Model with iid noise

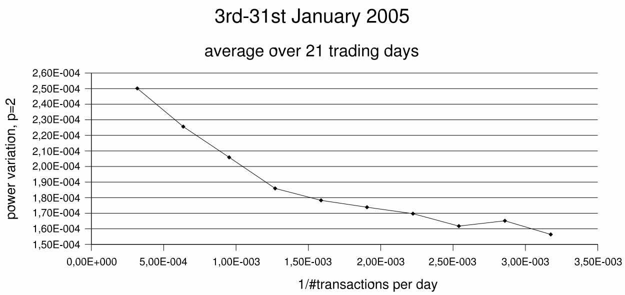
(cf. Ait-Sahalia, Mykland and Zhang (2006))

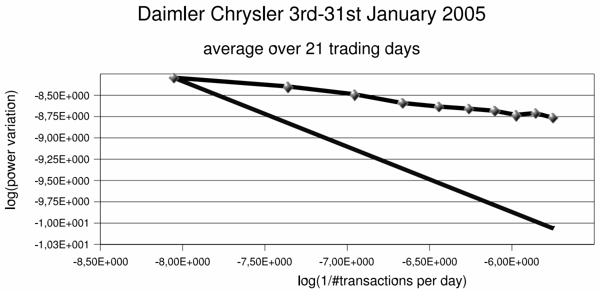
$$X_t = \int_0^t \sigma_s dB_s + \epsilon_t,$$

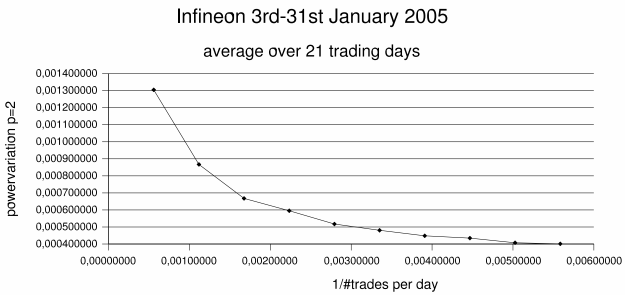
where ϵ denotes iid noise.
Then the realized volatility is of the order

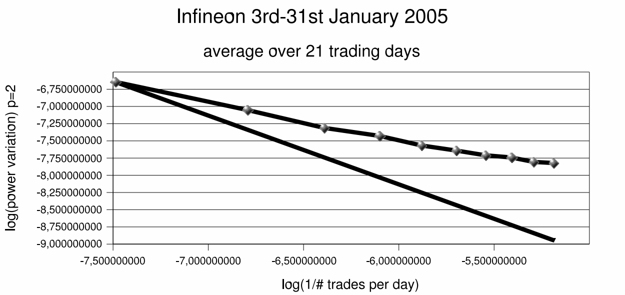
$$2\Delta^{-1}E(\epsilon^2),$$

hence the noise term leads to a bias with dominates the quadratic variation estimate for small Δ.


3rd-31st January 2005
average over 21 trading days
power variation, p=2
2,60E-004
2,50E-004
2,40E-004
2,30E-004
2,20E-004
2,10E-004
2,00E-004
1,90E-004
1,80E-004
1,70E-004
1,60E-004
1,50E-004
0,00E+000
5,00E-004
1,00E-003
1,50E-003
2,00E-003
2,50E-003
3,00E-003
3,50E-003
1/#transactions per day


Daimler Chrysler 3rd-31st January 2005
average over 21 trading days
log(power variation)
-8,50E+000
-8,75E+000
-9,00E+000
-9,25E+000
-9,50E+000
-9,75E+000
-1,00E+001
-1,03E+001
-8,50E+000
-8,00E+000
-7,50E+000
-7,00E+000
-6,50E+000
-6,00E+000
log(1/#transactions per day)


Infineon 3rd-31st January 2005
average over 21 trading days
powervariation p=2
0,001400000
0,001300000
0,001200000
0,001100000
0,001000000
0,000900000
0,000800000
0,000700000
0,000600000
0,000500000
0,000400000
0,00000000
0,00100000
0,00200000
0,00300000
0,00400000
0,00500000
0,00600000
1/#trades per day


Infineon 3rd-31st January 2005
average over 21 trading days
log(power variation) p=2
-6,750000000
-7,000000000
-7,250000000
-7,500000000
-7,750000000
-8,000000000
-8,250000000
-8,500000000
-8,750000000
-9,000000000
-7,500000000
-7,000000000
-6,500000000
-6,000000000
-5,500000000
log(1/# trades per day)

Non-normed and normed power variation

$$V_p^n(\int_0^t \sigma_s dB_s) \quad = \quad$$

$$\sum_i |\int_{t_{i-1}}^{t_i} \sigma_s dB_s|^p \quad \xrightarrow{p} \quad \begin{cases} 0 & : \quad p > 2 \\ \int_0^t \sigma_s^2 ds & : \quad p = 2 \\ \infty & : \quad p < 2 \end{cases}$$

$$\Delta^{1-p/2} V_p^n(\int_0^t \sigma_s dB_s) \xrightarrow{p} \mu_p \int_0^t \sigma_s^p ds,$$

as $n \to \infty$, where $\mu_p = E(|u|^p)$ with $u \sim N(0,1)$ and $\Delta = t_i - t_{i-1}$. (cf. Barndorff-Nielsen and Shephard (2003))

Power variation for the model with iid noise

Using Minkowski's inequality with $p > 1$ we obtain

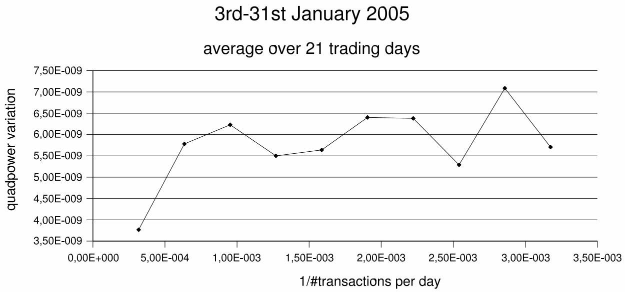
$$(\sum |\epsilon_{t_i} - \epsilon_{t_{i-1}}|^p)^{1/p} - (\sum |\int_{t_{i-1}}^{t_i} \sigma_s dB_s|^p)^{1/p} \leq (\sum |X_{t_i} - X_{t_{i-1}}|^p)^{1/p}$$

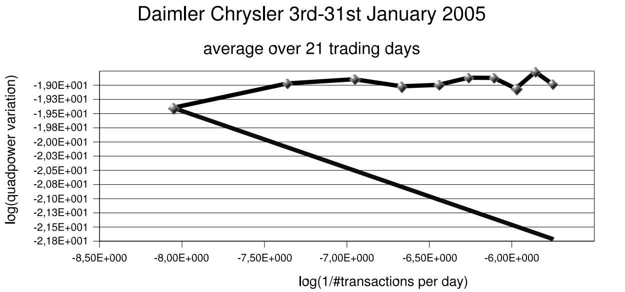
$$\leq (\sum |\epsilon_{t_i} - \epsilon_{t_{i-1}}|^p)^{1/p} + (\sum |\int_{t_{i-1}}^{t_i} \sigma_s dB_s|^p)^{1/p}$$

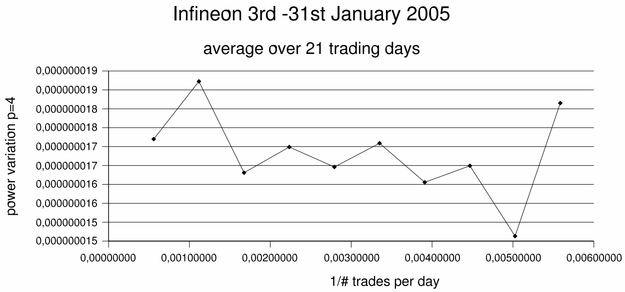
Hence if $E|\epsilon|^p < \infty$ for some $p > 2$, then

$$\sum |X_{t_i} - X_{t_{i-1}}|^p \to \infty,$$

which does not coincide with empirical findings.


3rd-31st January 2005
average over 21 trading days
quadpower variation
7,50E-009
7,00E-009
6,50E-009
6,00E-009
5,50E-009
5,00E-009
4,50E-009
4,00E-009
3,50E-009
0,00E+000
5,00E-004
1,00E-003
1,50E-003
2,00E-003
2,50E-003
3,00E-003
3,50E-003
1/#transactions per day


Daimler Chrysler 3rd-31st January 2005
average over 21 trading days
log(quadpower variation)
-1,90E+001
-1,93E+001
-1,95E+001
-1,98E+001
-2,00E+001
-2,03E+001
-2,05E+001
-2,08E+001
-2,10E+001
-2,13E+001
-2,15E+001
-2,18E+001
-8,50E+000
-8,00E+000
-7,50E+000
-7,00E+000
-6,50E+000
-6,00E+000
log(1/#transactions per day)


Infineon 3rd -31st January 2005
average over 21 trading days
power variation p=4
0,000000019
0,000000019
0,000000018
0,000000018
0,000000017
0,000000017
0,000000016
0,000000016
0,000000015
0,000000015
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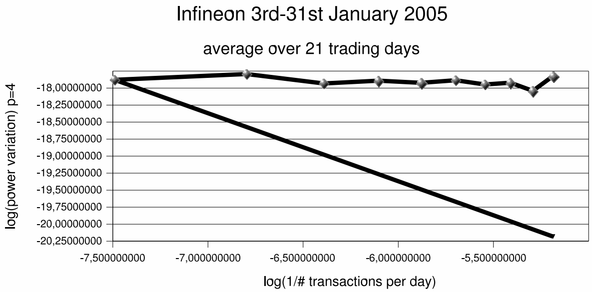
0,00100000
0,00200000
0,00300000
0,00400000
0,00500000
0,00600000
1/# trades per day


Infineon 3rd-31st January 2005
average over 21 trading days
log(power variation) p=4
-18,00000000
-18,25000000
-18,50000000
-18,75000000
-19,00000000
-19,25000000
-19,50000000
-19,75000000
-20,00000000
-20,25000000
-7,500000000
-7,000000000
-6,500000000
-6,000000000
-5,500000000
log(1/# transactions per day)

Fractional Brownian Motion

A fractional Brownian motion (fBm) with **Hurst parameter** $H \in (0,1)$, $B^H = \{B_t^H, t \geq 0\}$ is a zero mean Gaussian process with the covariance function

$$E(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}), \quad s,t \geq 0.$$

The fBm is a **self-similar** process, that is, for any constant $a > 0$, the processes
$\{a^{-H} B_{at}^H, t \geq 0\}$ and $\{B_t^H, t \geq 0\}$ have the same distribution.
For $H = \frac{1}{2}$, B^H coincides with the classical Brownian motion. For $H \in (\frac{1}{2}, 1)$ the process possesses **long memory** and for $H \in (0, \frac{1}{2})$ the behaviour is chaotic.

Non-normed Power Variation for fractional Brownian motion

$$\sum_i \left| \int_{t_{i-1}}^{t_i} \sigma_s dB_s^H \right|^p \xrightarrow{p} \begin{cases} 0 & : \quad p > 1/H \\ \mu_{1/H} \int_0^t \sigma_s^{1/H} ds & : \quad p = 1/H \\ \infty & : \quad p < 1/H \end{cases},$$

The integral is a **pathwise Riemann-Stieltjes integral** and we need that σ is a stochastic process with paths of finite q-variation, $q < \frac{1}{1-H}$.
Idea: Empirical behaviour of tick-by-tick data may also be explained by fBB with $H < 0.5$.

Consistency

joint with J.M. Corcuera and D. Nualart (2006)

Theorem

Suppose that σ_t is a stochastic process with finite q-variation, where $q < \frac{1}{1-H}$. Set

$$Z_t = \int_0^t \sigma_s dB_s^H.$$

Then,

$$\Delta^{1-pH} V_p^n(Z) \xrightarrow{P} \mu_p \int_0^T |\sigma_s|^p ds,$$

as $n \to \infty$.

Estimate for quadratic variation

We can explain the empirical findings by considering

$$\sum_i |X_{t_i} - X_{t_{i-1}}|^2 = \Delta^{2H-1}(\Delta^{1-2H}\sum_i |X_{t_i} - X_{t_{i-1}}|^2),$$

where

$$\Delta^{1-2H}\sum_i |X_{t_i} - X_{t_{i-1}}|^2 \xrightarrow{p} \int_0^t \sigma_s^2 ds$$

and $\Delta^{2H-1} \to \infty$ for $H < 0.5$.

More details:

We look at the test statistics:

$$S = \frac{\sum_{i=1}^{[nT]-1}(X_{\frac{i+1}{n}} - X_{\frac{i}{n}})(X_{\frac{i}{n}} - X_{\frac{i-1}{n}})}{\sum_{i=1}^{[nT]}(X_{\frac{i}{n}} - X_{\frac{i-1}{n}})^2}$$

model based on Brownian motion: 0
model based on Brownian motion with iid noise: -1/2
model based on fractional Brownian motion: $C_H = \frac{1}{2}(2^{2H} - 2)$
confidence interval for Brownian motion based model:

$$\left[-c_\gamma \sqrt{\frac{\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4}{3\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}}, c_\gamma \sqrt{\frac{\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4}{3\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}} \right],$$

where c_γ denotes the γ-quantile of a $N(0,1)$-distributed random variable.

Problems:

$\sum_{i=1}^{[nT]}(X_{\frac{i}{n}} - X_{\frac{i-1}{n}})^2$ is not robust to jumps.
Namely

$$\sum_{i=1}^{[nT]}(X_{\frac{i}{n}} - X_{\frac{i-1}{n}})^2 \to \int_0^T \sigma_s^2 ds + \sum_{s\leq T}(X_s - X_{s-})^2$$

Hence for $H < 0.5$ the denominator in S tends to infinity, which means our result might look like a Brownian motion based model even if it is not the case.

Replace it by bipower variation

$$n^{-1+2H} \sum_{i=1}^{[nT]-1} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}||X_{\frac{i+1}{n}} - X_{\frac{i}{n}}| \xrightarrow{p} K_H \int_0^t \sigma_s^2 ds,$$

with $K_H = E(|B_2 - B_1||B_1 - B_0|) = \frac{2}{\pi}(C_H \arcsin(C_H) + \sqrt{1 - C_H^2})$.
jointly with Barndorff-Nielsen, Corcuera and Podolskij (2008).

Some modification:

$$R = \frac{\sum_{i=1}^{[nt]-1} (X_{\frac{i+1}{n}} - X_{\frac{i}{n}})(X_{\frac{i}{n}} - X_{\frac{i-1}{n}})}{\sum_{i=1}^{[nt]-1} |X_{\frac{i+1}{n}} - X_{\frac{i}{n}}||X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|} \xrightarrow{p} \frac{C_H}{K_H},$$

where $K_H = E(|B_2 - B_1||B_1 - B_0|) = \frac{2}{\pi}(C_H \arcsin(C_H) + \sqrt{1 - C_H^2})$

$$\left[\frac{C_H}{K_H} - c_\gamma \sqrt{\frac{v^2 \sum_{i=1}^{[nt]-2} |X_{\frac{i+2}{n}} - X_{\frac{i+1}{n}}|^{4/3} |X_{\frac{i+1}{n}} - X_{\frac{i}{n}}|^{4/3} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^{4/3}}{\mu_{4/3}^3 \left(\sum_{i=1}^{[nt]-1} |X_{\frac{i+1}{n}} - X_{\frac{i}{n}}||X_{\frac{i}{n}} - X_{\frac{i-1}{n}}| \right)^2}}, \right.$$

$$\left. \frac{C_H}{K_H} + c_\gamma \sqrt{\frac{v^2 \sum_{i=1}^{[nt]-2} |X_{\frac{i+2}{n}} - X_{\frac{i+1}{n}}|^{4/3} |X_{\frac{i+1}{n}} - X_{\frac{i}{n}}|^{4/3} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^{4/3}}{\mu_{4/3}^3 \left(\sum_{i=1}^{[nt]-1} |X_{\frac{i+1}{n}} - X_{\frac{i}{n}}||X_{\frac{i}{n}} - X_{\frac{i-1}{n}}| \right)^2}} \right].$$

Remark:

- Strictly speaking the distributional theory only works for $H < 0.5$ and $\beta \leq 1$ as has been shown in the context of multipower variation for Brownian semimartingales.
- Furthermore, multipower variation is less efficient than power variation.

Daimler Chrysler, January 3rd-31st 2005, 1% level

# transactions	mean distance	S	l. bound BM
66140	7s	-0.1061	-0.0796
33070	14s	-0.1606	-0.1094
22046	21s	-0.1574	-0.1244
16535	28s	-0.1192	**-0.1295**
13228	35s	-0.1156	**-0.1367**

# transactions	mean distance	R	l. bound BM
66140	7s	-0.424	-0.0357
33070	14s	-0.4411	-0.0405
22046	21s	-0.3837	-0.0622
16535	28s	-0.2744	-0.0467
13228	35s	-0.2532	-0.0518
11023	42s	-0.2202	-0.0597
9448	49s	-0.1749	-0.0601
8267	56s	-0.1117	-0.0675
7348	63s	-0.1336	-0.0623
6614	70s	-0.0972	-0.0702

Model with market microstructure

Confidence interval for Brownian motion based model with iid noise:

$$\left[-\frac{1}{2} - c_\gamma \sqrt{\frac{\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4 - \frac{1}{2n}\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}{2\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}},\right.$$

$$\left.-\frac{1}{2} + c_\gamma \sqrt{\frac{\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4 - \frac{1}{2n}\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}{2\left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2\right)^2}}\right],$$

where c_γ denotes the γ-quantile of a standard normal random variable.

Model with market microstructure

Daimler Chrysler, January 3rd-31st 2005, 1% level

# transactions	mean distance	S	u. bound iid
66140	7s	-0.1061	-0.4026
33070	14s	-0.1606	-0.3661
22046	21s	-0.1574	-0.3477
16535	28s	-0.1192	-0.3415
13228	35s	-0.1156	-0.3326
11023	42s	-0.0994	-0.3171
9448	49s	-0.0827	-0.313
8267	56s	-0.0542	-0.3213
7348	63s	-0.0608	-0.2974
6614	70s	-0.0487	-0.3081
661	700s	-0.0861	-0.1387

Fractional Brownian motion based model

confidence interval for $H < 3/4$:

$$\left[C_H - c_\gamma \sqrt{\frac{V^2 \sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4}{\mu_4 \left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2 \right)^2}}, \right.$$

$$\left. C_H + c_\gamma \sqrt{\frac{V^2 \sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^4}{\mu_4 \left(\sum_{i=1}^{[nt]} |X_{\frac{i}{n}} - X_{\frac{i-1}{n}}|^2 \right)^2}} \right],$$

where c_γ denotes the γ-quantile of a standard normal random variable and we use the following notation

$V^2 = v^2 - C_H(8\rho_H(1) + 8\sum_{j\geq 1} \rho_H(j)\rho_H(j+1)) + C_H^2 v_1^2$

$v^2 = \frac{1}{4}\tilde{v}_1^2 - v_1^2 - 8\rho_H(1) - 8\sum_{j\geq 1} \rho_H(j)\rho_H(j+1)$ and

$\tilde{v}_1^2 = 2 \times 2^{4H} + 4\sum_{j=1}^{\infty} \tilde{\rho}_H(j)^2$

$v_1^2 = 2 + 4\sum_{j=1}^{\infty} \rho_H(j)^2$, $\tilde{\rho}_H(j) = \frac{1}{2}((j+2)^{2H} + |j-2|^{2H} - 2j^{2H})$,

$\rho_H(j) = \frac{1}{2}((j+1)^{2H} + (j-1)^{2H} - 2j^{2H})$.

Estimation of H

Use S to estimate H by inverting C_H.
Empirical results for fractional Browninan motion (cf. Xing (2008))
$n = 1024$

H	0.25	$1/3$	0.35	0.45	0.55	0.65
mean	0.249	0.3324	0.3491	0.4492	0.5494	0.6494
variance	0.0008	0.0007	0.0007	0.0006	0.0005	0.0004

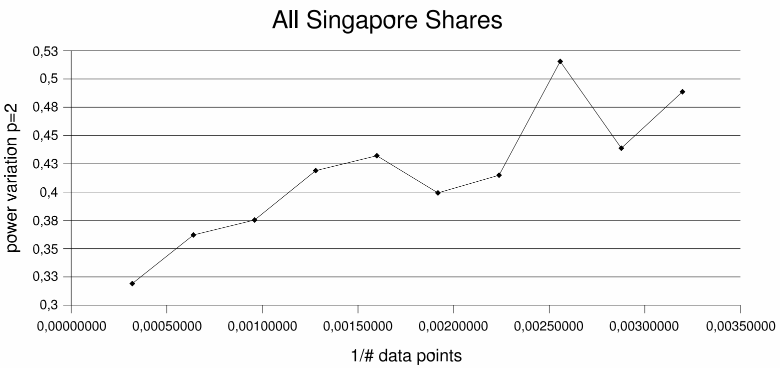
What are the effects of these results?

Risk induced by model misspecification
We look at Daimler Chrysler data of 12.1.2005:
Assuming a model based on **Brownian motion**:

$$\int_0^T \sigma_s^2 ds = 0.000309$$

Assuming a model based on **fractional Brownian motion** with $H = 0.4$:

$$\int_0^T \sigma_s^2 ds = 0.000059$$


All Singapore Shares
power variation p=2
0,53
0,5
0,48
0,45
0,43
0,4
0,38
0,35
0,33
0,3
0,00000000
0,00050000
0,00100000
0,00150000
0,00200000
0,00250000
0,00300000
0,00350000
1/# data points

What are the effects of these results?

Risk induced by model misspecification

We look at index data from Singapore:

Assuming a model based on **Brownian motion**:

$$\int_0^T \sigma_s^2 ds = 0.319$$

Assuming a model based on **fractional Brownian motion** with $H = 0.6$:

$$\int_0^T \sigma_s^2 ds = 1.595$$

VaR

Calculation of 10-day-VaR from the 1-day-VaR:

Factor $10^{0.6} = 3.98$ instead of $10^{0.5} = 3.16$

Do we have an additional jump component?

A measure for the **activity** of the jump component of a Lévy process is the **Blumenthal-Getoor** index β,

$$\beta = \inf\{\delta > 0 : \int (1 \wedge |x|^{\delta})\nu(dx) < \infty\}.$$

This index ensures, that for $p > \beta$ the sum of the p-th power of jumps will be finite.

Comparison of non-normed power variation

$$\sum_i \left| \int_{t_{i-1}}^{t_i} \sigma_s dB_s \right|^p \xrightarrow{p} \begin{cases} 0 & : \quad p > 2 \\ \int_0^t \sigma_s^2 ds & : \quad p = 2 \\ \infty & : \quad p < 2 \end{cases}$$

and the case for the Lévy model

$$\sum_i \left| \int_{t_{i-1}}^{t_i} \sigma_s dL_s \right|^p$$

$$\xrightarrow{p} \begin{cases} \sum(\left| \int_{u-}^{u} \sigma_s dL_s \right|^p : 0 < u \leq t) & : \quad p > \beta \\ \infty & : \quad p < \beta \end{cases}$$

under appropriate regularity conditions, where β denotes the Blumenthal-Getoor index of L.

Non-normed Power Variation for fractional Brownian motion

$$\sum_i |\int_{t_{i-1}}^{t_i} \sigma_s dB_s^H|^p \xrightarrow{p} \begin{cases} 0 & : \quad p > 1/H \\ \mu_{1/H} \int_0^t \sigma_s^{1/H} ds & : \quad p = 1/H \\ \infty & : \quad p < 1/H \end{cases},$$

where $H > 1/2$. The integral is a **pathwise Riemann-Stieltjes integral** and we need that σ is a stochastic process with paths of finite q-variation, $q < \frac{1}{1-H}$.
Hence one over the **Hurst exponent** plays a similar role as the **Blumenthal-Getoor index**.

Log-Power Variation Estimators

Theorem

Assume that for some $k \in \mathbb{R}$ and $p \in (a, b)$, s.t. $1 - pk \neq 0$

$$\Delta^{1-pk} V_p^n(X) \xrightarrow{p} C, \tag{1}$$

with $0 < C < \infty$, then

$$\frac{\ln(\Delta V_p^n(X))}{p \ln \Delta} \xrightarrow{p} k \tag{2}$$

holds as $n \to \infty$, if on the other hand

$$V_p^n(X) \xrightarrow{p} C, \tag{3}$$

with $0 < C < \infty$, then as $n \to \infty$

$$\frac{\ln(\Delta V_p^n(X))}{p \ln \Delta} \xrightarrow{p} \frac{1}{p}. \tag{4}$$

Question:

When is condition (1) or (3) satisfied

The definition of the Blumenthal-Getoor index for $p > \beta$ yields (3).

(1) has been considered in the framework of estimating the **integrated volatility** for many models :

- classical stochastic volatility models based on Brownian motion with general mean process and additional jump component.
- models based on fractional Brownian motion.
- models based on Lévy processes.

Result: fractional Brownian motion

$$\Delta_n^{1-pH} V_p(X, S_n) \xrightarrow{p} E|V|^p \int_0^t \sigma_s^p ds,$$

where $V \sim N(0,1)$ and $p > 0$.
Hence we obtain (1) with:

$$C = \text{const.} \int_0^t \sigma_u^p du$$

$k = H$ and $p > 0$.

Result:
Lévy processes

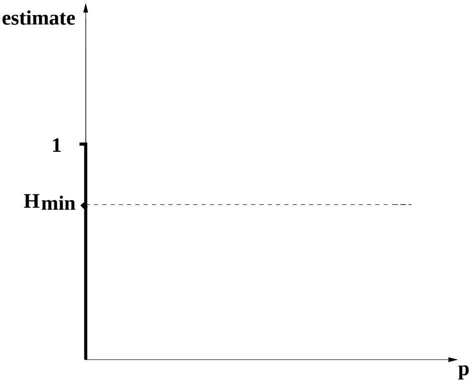
$$\Delta_n^{1-\frac{p}{\beta}} V_p(X, S_n) \xrightarrow{p} c_L E|U|^p \int_0^t \sigma_s^p ds,$$

where U denotes a β-stable random variable, c_L a constant, depending on L and $p < \beta$.
Hence we obtain (1) with:

$$C = \text{const.} \int_0^t \sigma_u^p du$$

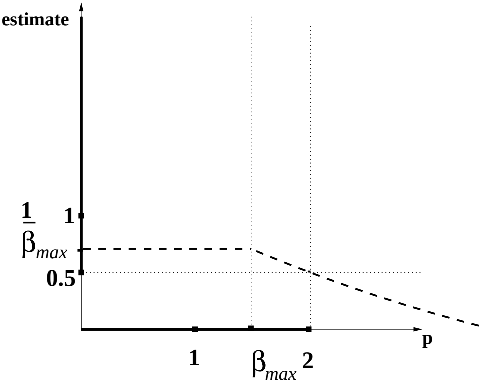
$k = 1/\beta$ and $p < \beta$.

Purely continuous model



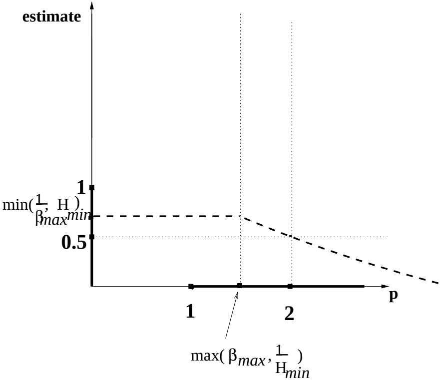
$$0 < H_{min} < 1$$

Pure jump model



$$\frac{1}{2} < \frac{1}{\beta_{max}} < \infty$$

Mixed model



$$0 < \beta_{max} < 2, \quad 0 < H_{min} < 1$$

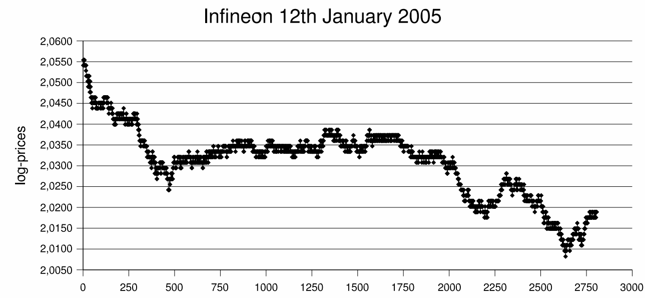
$$0 < \min(\frac{1}{\beta_{max}}, H_{min}) < 1, \quad 1 < \max(\beta_{max}, \frac{1}{H_{min}}) < \infty$$

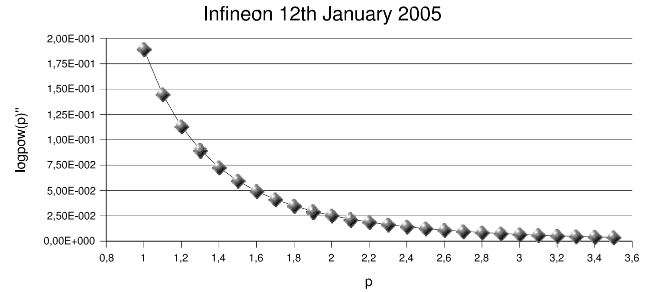
How to determine a jump component

Look at the behaviour of the **second derivative** of the log-power variation in p.

Example 1: Infineon data
12th January 2005: 2806 transactions
26th January 2005: 1977 transactions

Example 2: Daimler Chrysler data
12th January 2005: 3960 transactions
26th January 2005: 3328 transactions


Infineon 12th January 2005
log-prices
2,0600
2,0550
2,0500
2,0450
2,0400
2,0350
2,0300
2,0250
2,0200
2,0150
2,0100
2,0050
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000


Infineon 12th January 2005
logpow(p)"
2,00E-001
1,75E-001
1,50E-001
1,25E-001
1,00E-001
7,50E-002
5,00E-002
2,50E-002
0,00E+000
0,8
1
1,2
1,4
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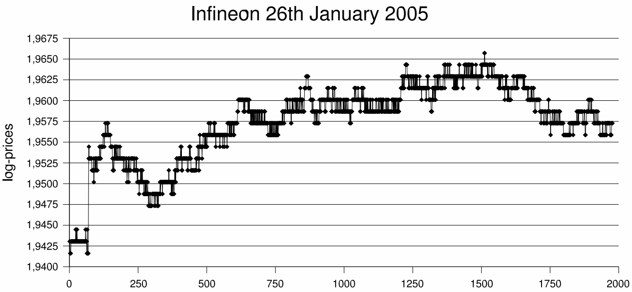
2,4
2,6
2,8
3
3,2
3,4
3,6
p


Infineon 26th January 2005
log-prices
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1,9650
1,9625
1,9600
1,9575
1,9550
1,9525
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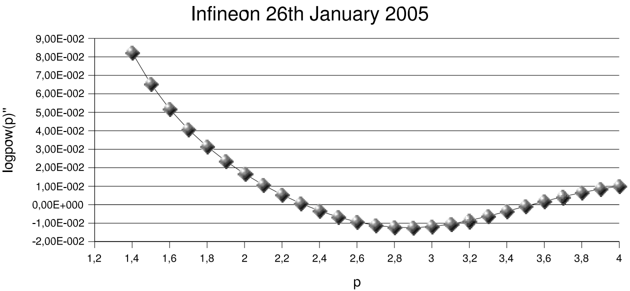
1,9450
1,9425
1,9400
0
250
500
750
1000
1250
1500
1750
2000


Infineon 26th January 2005
logpow(p)"
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8,00E-002
7,00E-002
6,00E-002
5,00E-002
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3,00E-002
2,00E-002
1,00E-002
0,00E+000
-1,00E-002
-2,00E-002
1,2
1,4
1,6
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3
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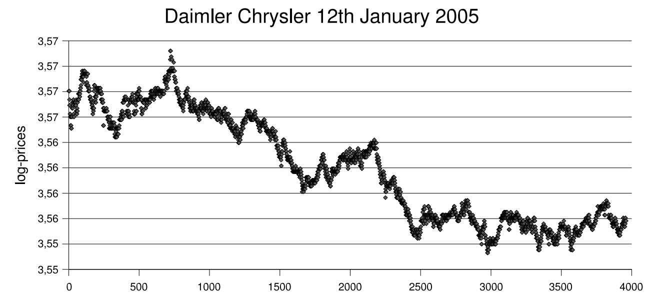
3,6
3,8
4
p


Daimler Chrysler 12th January 2005
log-prices
3,57
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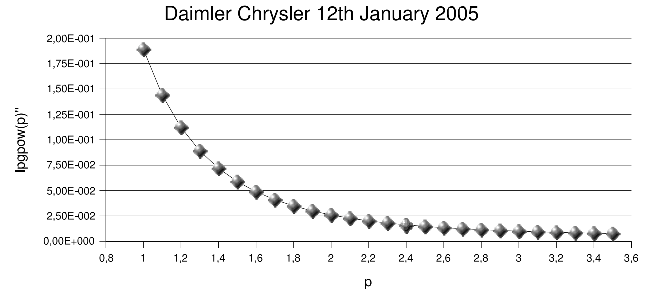
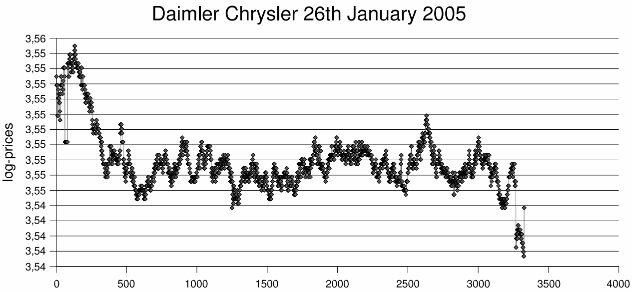
3,57
3,57
3,56
3,56
3,56
3,56
3,55
3,55
0
500
1000
1500
2000
2500
3000
3500
4000


Daimler Chrysler 12th January 2005
lpgpow(p)"
2,00E-001
1,75E-001
1,50E-001
1,25E-001
1,00E-001
7,50E-002
5,00E-002
2,50E-002
0,00E+000
0,8
1
1,2
1,4
1,6
1,8
2
2,2
2,4
2,6
2,8
3
3,2
3,4
3,6
p


Daimler Chrysler 26th January 2005
log-prices
3,56
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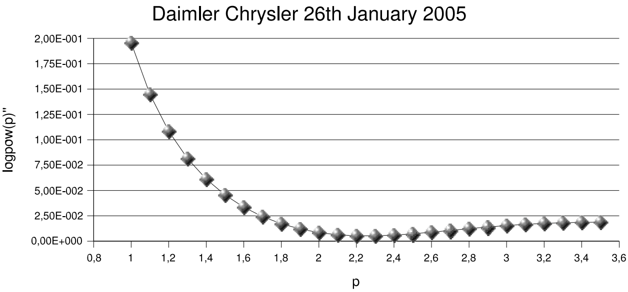
0
500
1000
1500
2000
2500
3000
3500
4000


Daimler Chrysler 26th January 2005
logpow(p)"
2,00E-001
1,75E-001
1,50E-001
1,25E-001
1,00E-001
7,50E-002
5,00E-002
2,50E-002
0,00E+000
0,8
1
1,2
1,4
1,6
1,8
2
2,2
2,4
2,6
2,8
3
3,2
3,4
3,6
p

Conclusion:

- Increasing limits in quadratic variation for data may be explained by fractional Brownian motion with $H < 0.5$.
- This approach may be applied to financial and climate data.
- log-power variation may be used to detect jump components in both Brownian and fractional Brownian motion based models.