

Solve Wave Speed

`TravelWaveEquation = ForAll[U, a * s == a^2 * D * (1 - 2 U) + A (U - α)]`

$\forall U, a, s = a^2 D (1 - 2 U) + A (U - \alpha)$

`? Resolve`

`Resolve[expr]` attempts to resolve `expr` into a form that eliminates `ForAll` and `Exists` quantifiers. `Resolve[expr, dom]` works over the domain `dom`. Common choices of `dom` are `Complexes`, `Reals` and `Booleans`. \gg

`ResolvedEqn = Resolve[ForAll[U, TravelWaveEquation]]`

$A - 2 a^2 D = 0 \ \&\& \ a^2 D - a s - A \alpha = 0$

`Soln_Las = Solve[ResolvedEqn, {a, s}] // Simplify`

$$\left\{ \left\{ s \rightarrow \frac{\sqrt{A} \sqrt{D} (1 - 2 \alpha)}{\sqrt{2}}, a \rightarrow \frac{\sqrt{A}}{\sqrt{2} \sqrt{D}} \right\}, \left\{ s \rightarrow \frac{\sqrt{A} \sqrt{D} (-1 + 2 \alpha)}{\sqrt{2}}, a \rightarrow -\frac{\sqrt{A}}{\sqrt{2} \sqrt{D}} \right\} \right\}$$

Quadrature for U(ξ)

`UIntegral = Integrate[1 / (a * U * (1 - U)), U] /. First[Soln_Las] // Simplify`

$$\frac{\sqrt{2} \sqrt{D} (-\text{Log}[-1 + U] + \text{Log}[U])}{\sqrt{A}}$$

`USoln = Solve[UIntegral == ξ + const, U] // Simplify`

$$\left\{ \left\{ U \rightarrow \frac{e^{\frac{\sqrt{A} (\text{const} + \xi)}{\sqrt{2} \sqrt{D}}}}{\sqrt{A} (\text{const} + \xi) - 1 + e^{\frac{\sqrt{A} (\text{const} + \xi)}{\sqrt{2} \sqrt{D}}}} \right\} \right\}$$

`USolnWithConstantChoice = Simplify[U /. USoln /. const -> I * Pi * Sqrt[2 * D / A], Assumptions -> {A > 0, D > 0}][[1]]`

$$\frac{e^{\frac{\sqrt{A} \xi}{\sqrt{2}}}}{1 + e^{\frac{\sqrt{A} \xi}{\sqrt{2}}}}$$

This solution goes from 0 to 1:

```
Plot[Exp[x] / (1 + Exp[x]), {x, -10, 10}]
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