Mathematical Modelling and Scientific Computing in the Biosciences II

Exercise 1 (Due in 2 weeks: Tues, 30 October)

The ODE system for pituary gonadotroph ER oscillator in closed cell is:

$$\frac{d\left[Ca^{2+}\right]_i}{dt} =$$

$$\frac{fi}{Vi} * \left(\left(L + P * \left(\frac{I * \left[Ca^{2+}\right]_i * h}{(I + Ki) * \left(\left[Ca^{2+}\right]_i + Ka \right)} \right)^3 \right) * \left(\left[Ca^{2+}\right]_E - \left[Ca^{2+}\right]_i \right) - Ve \frac{\left[Ca^{2+}\right]_i^2}{Ke^2 + \left[Ca^{2+}\right]_i^2} + \in * \left(Jin - Vp \frac{\left[Ca^{2+}\right]_i^2}{Kp^2 + \left[Ca^{2+}\right]_i^2} \right) \right)$$

$$\frac{dh}{dt} = A * \left(Kd - \left(\left[Ca^{2+}\right]_i + Kd \right) * h \right)$$

$$\frac{d\left[Ca^{2+}\right]_T}{dt} = \frac{fi}{Vi} * \in * \left(Jin - Vp \frac{\left[Ca^{2+}\right]_i^2}{\left(Kp^2 + \left[Ca^{2+}\right]_i^2 \right)} \right)$$

where

$$\left[Ca^{2+}\right]_E = 1 / \sigma * \left(\left[Ca^{2+}\right]_T - \left[Ca^{2+}\right]_i \right)$$

Take parameter values

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fi = 0.01; Vi = 4; L = 0.37; P = 26640; I = 0.9;
Ki = 1.0; Ka = 0.4; Ve = 400; Ke = 0.2; A = 0.5;
Kd = 0.4; σ = 0.185; ∈ = 0.01; Vp = 2000; Kp = 0.3;
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and initial conditions

$$\left[Ca^{2+}\right]_i(0) = 0.2,\ h(0) = 0.8,\ \left[Ca^{2+}\right]_T(0) = 4$$

- Question 1 (8 points): integrate the ODE system numerically, with Jin = 1200 if t <= 40, Jin = 0 for t > 40. What do you observe in $\left[Ca^{2+}\right]_T$?
- Question 2 (2 points): for small ϵ, what can be extracted as the slow variable of the system?
- Question 3 (10 points): hence, plot bifurcation diagram with the slow variable as the bifurcation parameter. Super-impose the time-series on it to show the behavior can be captured in terms of the bifurcation diagram.