Kinetic Wave in Immature Xenopus Oocyte

Ex3: Due Friday, 8 Feb 2008

Equations

As was given in the lecture, the PDE system is of the form:

```
{ ∂t c[t, x] == CoeffD * ∂x,x c[t, x] +
     f[c[t, x], w[t, x]],
  ∂t w[t, x] == g[c[t, x], w[t, x]]};
```

where the functions `f`, `g` are defined as follows:

- **Vector field for gating variable, w**

```
In[37]:= Q2 = d2 * (IP3 + d1) / (IP3 + d3);
wInf[c_] := Q2 / (Q2 + c);
Tau[c_] := 1 / (a2 * (Q2 + c));
g[c_, w_] := (wInf[c] - w) / Tau[c]  ;
```

- **Vector field for Calcium, $[Ca^{2+}]_i$**

```
In[41]:= mInf[c_] := (IP3 / (IP3 + d1) * c / (c + d5));
VIP3R[c_, w_] := (mInf[c] * w) ^3;
f[c_, w_] :=
  (fiVLeak + fiVIP3R * VIP3R[c, w]) * (cER - c)
  fiVSERCA * c ^ 2 / (c ^ 2 + KSerca ^ 2);
```

and where we choose the following parameters:

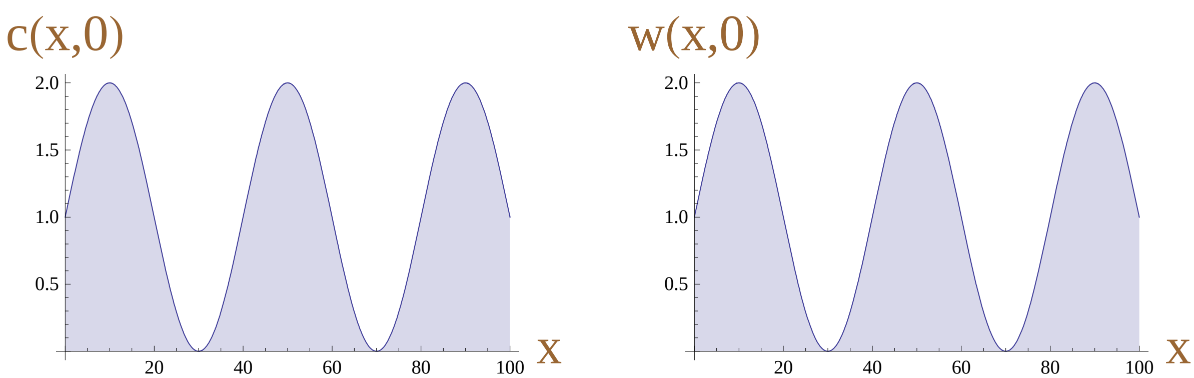
```
In[44]:= ParamRules = {fiVLeak → 0.004, fiVIP3R → 20, fiVSERCA → 1.2, cER → 1,
   KSerca → 0.15, IP3 → 0.7, a2 → 0.2, d1 → 0.1, d2 → 0.5, d3 → 0.2, d5 → 0.15};
```

Let us now choose `CoeffD` = 0.05 (low value) and the following initial conditions for `c` and `w`

Note that we want the initial condition to be spatially periodic over the domain, hence we choose `xRange` to be some multiple of `DesiredWavePeriod`

In[176]:=
```
DesiredWavePeriod = 20;

(* set variable range for plotting and solution *)
xRange = {x, 0, 100 };
tRange = {t, 0, 200};

InitialConditionProfileC = (1 + Sin[ Pi * x / DesiredWavePeriod]);
InitialConditionProfileW = (1 + Sin[ Pi * x / DesiredWavePeriod]);

CInitPlot = Plot[InitialConditionProfileC, {x, 0, xRange // Last}, PlotRange → All,
   Filling → Bottom,
   AxesLabel → {StyleForm["x", Large, Brown], StyleForm["c(x,0)", Large, Brown]}];

WInitPlot = Plot[InitialConditionProfileW, {x, 0, xRange // Last}, PlotRange → All,
   Filling → Bottom,
   AxesLabel → {StyleForm["x", Large, Brown], StyleForm["w(x,0)", Large, Brown]}];

Show[GraphicsArray[{CInitPlot, WInitPlot}], ImageSize → {900, 350}]
```

Out[183]=



Q1 (10 points): Using the above initial conditions, solve the given PDE system, with diffusion coefficient = 0.05 and with periodic boundary conditions. Do density plots of the solutions `c` and `w`. Hint: to compute and plot the solution, you can refer to the notebook as given in lecture.

Q2 (5 points): repeat the above with `DesiredWavePeriod` = 50, note the similarity of behavior as above.

Kinetic Wave: Wave-Speed Dependence on Initial Condition

In the case that the diffusion coupling is small, one can in fact get waves with speeds that depends on the particular form of initial conditions. These are the characteristics of *kinetic waves*, as opposed to the bistable, *travelling wave*.

In particular, we will now look at the situation where one chooses an initial condition that is spatially periodic, i.e., $c_0(x+\lambda) = c_0(x)$. If in addition, $c_0(x)$ and $w_o(x)$ are chosen to be a *limit cycle solution* for the ODE system (i.e., obtained from the PDE system by ignoring the spatial diffusion term), then we will in fact have: $c(x+\lambda, t) = c(x,t)$, $w(x+\lambda, t) = w(x,t)$.

In Question 3, we will set up such an initial condition profile. From the PDE solution (with small diffusion), we will see that c(x,t) = c(x+v t), where $v \approx \frac{\lambda}{\tau}$ with τ being the period of limit cycle f oscillation or the ODE solution.

Q3 (25 points): construct 2 sets of initial conditions (for $c_0(x)$, $w_o(x)$) that both lie on the limit cycle of the ODE system

```
c'[t] == f[c[t], w[t]]
w'[t] == g[c[t], w[t]]
```

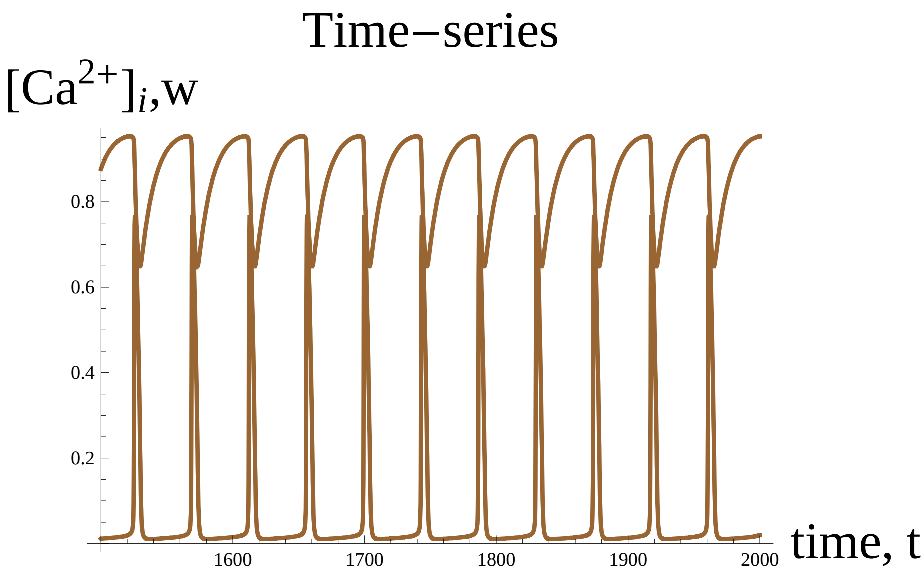
but with the (spatial) periods 25 and 100 respectively (hint: use scaling as was done in setting up Q1, via `DesiredWavePeriod`). Then, demonstrate kinetic waves using diffusion coefficient = 0.05.

You can either do this on your own, or following the steps suggested below.

- **Q3.1**

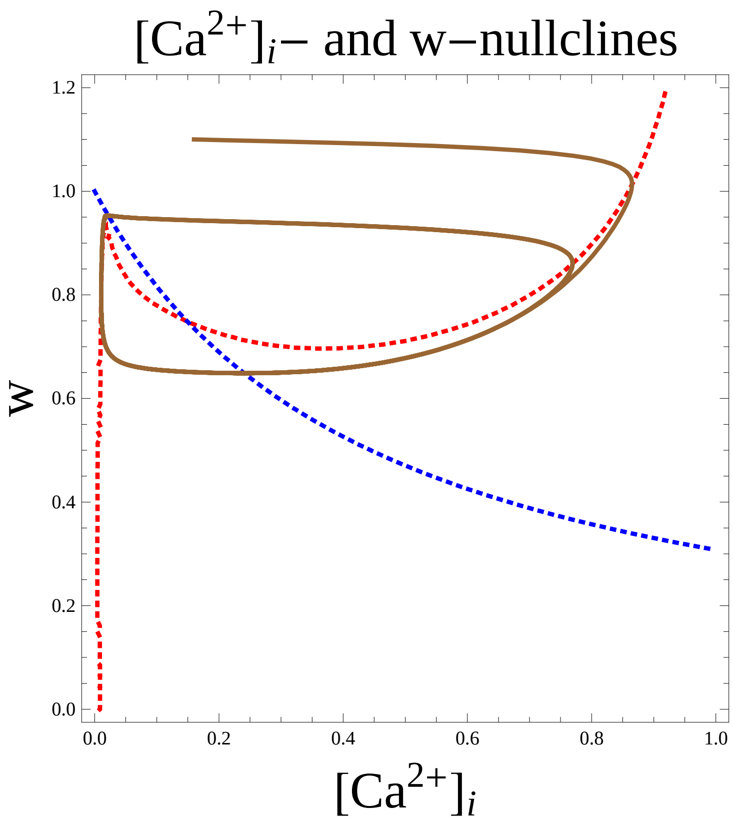
Solve the ODE system above to **tEnd**=2000, with initial conditions w[0] == 1.1, c[0] == 0.16

If you ignore the initial transient (i.e., say up to t=1500), you should get a numerical approximation to the limit cycle looking like below:



One can get a feel for the time-series by superimposing on the nullclines of the 2-dimensional ODE system:

Out[281]=



- **Q3.2**

We then need to compute the period for the above computed limit cycle solution.

This can be done numerically by finding the peaks for c, and then find the times at which the peaks occur.
We can use the commands `NMaximize` and `FindRoot`

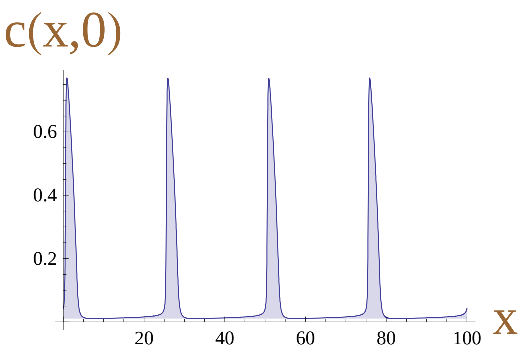
Hint: use commands of the form `cMax = NMaximize[{(c[t] /. Flatten[ODESoln]), t <= tEnd, t >= 1500}, t]`

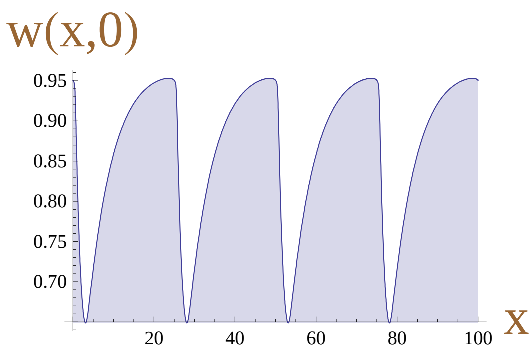
and then do a `FindRoot` for value of t attaining this, around the points t=1700 and t=1740.

The temporal period τ should be around 45.

- **Q3.3**

Transform the initial conditions to have spatial period of 25, so that if you plot them they look something like this:





Hint: you may do this using a command that involves the replacement rule: `t → (x + tEnd/4)*τ/DesiredWavePeriod`

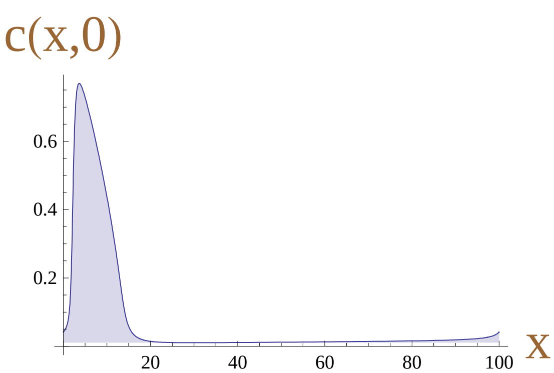
- **Q3.4**

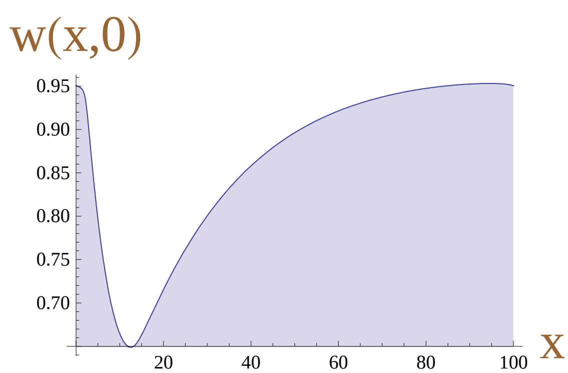
Using the above initial condition, solve the PDE system and do a density plot of the solution.
The solution should look translationally invariant.

- **Q3.5**

Repeat the process, using the folowing initial condition, with DesiredWavePeriod = 100:

Out[244]=





You should observe wavespeeds that look quite different compared to that of Q3.4.