POWER GEOMETRY AS NEW MATHEMATICS

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Power Geometry is a new calculus developing the differential calculus and aimed at the nonlinear problems. Its main concept consists in the study of nonlinear problems not in the original coordinates, but in the logarithms of these coordinates. Then some linear relations can be put in correspondence to many properties and relations, which are nonlinear in the original coordinates. The algorithms of Power Geometry are based on these linear relations. They allow to simplify equations, to resolve their singularities (including singular perturbations), to isolate their first approximations, and to find either asymptotic behaviors or asymptotic expansions of their solutions [1,2].

Algorithms of Power Geometry are applicable to equations of various types: algebraic, ordinary differential and partial differential, and also to systems of such equations. These algorithms include the simplifying transformations of coordinates and trucations of equations. Power Geometry is an alternative to Algebraic Geometry, Differential Algebra, Group Analysis, Nonstandard Analysis, and other disciplines.

Applications of Power Geometry were in problems of Mathematics (expansions of solutions to general ODEs [3,4] and to Painleve equations [5]), of Mechanics (motion of a rigid body [6]), of Celestial Mechanics (rotation of a satellite [7,8] and the restricted three-body problem [9]), of Hydromechanics (the boundary layer on a needle [10,11]), and in questions of integrability [12–14] and stability [15].

Only a few algorithms of Power Geometry were implemented.

REFERENCES (A.D. Bruno et all.)

1. Local Methods in Nonlinear Differential Equations. Springer-Verlag: Berlin–Heidelberg, 1989. 350 p.

2. Power Geometry in Algebraic and Differential Equations. Elsevier, Amsterdam, 2000. 395 p.

3. Asymptotic behaviors and expansions of solutions to an ordinary differential equation // Uspekhi Matem. Nauk 59:3 (2004) 31–80 = Russian Mathem. Surveys 59:3 (2004) 429–480.

4. Complicated expansions of solutions to an ordinary differential equation // Doklady Akademii Nauk 406:6 (2006) 730–733 = Doklady Mathematics 73:1 (2006) 117–120.

5. Expansions of solutions to the fifth Painleve equation (with E.S. Karulina) // Doklady Akademii Nauk 395:4 (2004) 439–444 = Doklady Mathematics 69:2 (2004) 214–220.

   Expansions of solutions to the sixth Painleve equation (with I.V. Goruchkina) // Ibid. 395:6 (2004) 733–737 = Ibid. 69:2 (2004) 268–272.

   Expansions of solutions to the sixth Painleve equation in cases a=0 and b=0 (with I.V. Goruchkina) // Doklady Akademii Nauk 410:3 (2006) 295–300 = Doklady Mathematics 74:2 (2006) 660–665.

6. Computation of power expansions of modified motions of a rigid body (with V.V. Lunev)// Doklady Akademii Nauk 386:1 (2002) 11–17 = Doklady Mathematics 66:2 (2002) 161–167.

   Families of power expansions of modified motions of a rigid body (with V.V. Lunev) // Doklady Akademii Nauk 387:3 (2002) 287–303 = Doklady Mathematics 66:3 (2002) 340–347.

   Power properties of motions of a rigid body // Ibid. 387:6 (2002) 727–732 = Ibid. 66:3 (2002) 415–420.

   Local integrability of the Euler-Poisson equations // Doklady Akademii Nauk 409:3 (2006) 295–299 = Doklady Mathematics 74:1 (2006) 512–516.

   Simple exact solutions to the N. Kowalewski equations (with I.N. Gashenenko) // Doklady Akademii Nauk 409:4 (2006) 439–442 = Doklady Mathematics 74:1 (2006) 536–539.

7. The limit problems for the equation of oscillations of a satellite (with V.P. Varin) // Celestial Mechanics and Dynamical Astronomy 67:1 (1997) 1–40.

8. Families of periodic solutions to the Beletsky equation // Kosmicheskie Issledovanija 40:3 (2002) 295–316 = Cosmic Research 40:3 (2002) 274–295.

9. The Restricted 3-Body Problem: Plane Periodic Orbits. Walter de Gruyter, Berlin-New York, 1994. 362 p.

10. On an axially symmetric flow of a viscous incompressible fluid around a needle (with T.V. Shadrina) // Ibid. 387:5 (2002) 589–595 = Ibid. 66:3 (2002) 396–402.

Axisymmetric boundary layer on a needle (with T.V. Shadrina) // Doklady Akademii Nauk 394:3 (2004) 298–304 = Doklady Mathematics 69:1 (2004) 57–63.

11. Axisymmetric boundary layer on a needle (with T.V. Shadrina) // Trudy Mosc. Mat. Obsch. 68 (2007) 226–290 = Trans. Moscow Math. Soc. 68 (2007) 226–290.

12. Invariant relations of the Fokker-Planck system (with V.V. Lunev) // Doklady Akademii Nauk 390:6 (2003) 733–739 = Doklady Mathematics 67:3 (2003) 416–422.

13. Normal forms and integrability of ODE systems (with V.F. Edneral) // Programmirovanie 32:3 (2006) = Programming and Computer Software 32:3 (2006) 139–144.

14. On integrability of the Euler-Poisson equations (with V.F. Edneral) // J. Calmet, W.M. Seiler, R.W. Tucker (Eds.): Global Integrability of Fields Theories, Universitaets-Verlag Karlsruhe, 2006, p. 39–56.

15. On computation of the Hamiltonian normal form (with A.G. Petrov) // Doklady Akademii Nauk 410:4 (2006) 474–478 = Doklady Physics 51:10 (2006) 555–559.

And a lot of KIAM preprints in Russian.