Attention: Please use the “Guideline for writing a proposal” when filling in this form. When this form (part 2 of the proposal) is completed it should only be about 14-19 pages.

1. Persons and organisational structure *(about 4-6 pages)*

Track record and profile of project manager: o.Univ.-Prof.Dr.Peter Markowich, Faculty of Mathematics, University of Vienna *(please fill in the name)*

Specific competencies for the project	25 years of research experience in nonlinear partial differential equations; Extensive photographic skills (see http://pbase.com/markowich); Wittgenstein Award of the Austrian Research Fund (FWF) 2000; Corresponding Member of the Austrian Academy of Sciences since 2005; Coordination and participation in Austrian and international grants and network projects in the area of nonlinear partial differential equations
Highlight of recent work	Entropy dissipation techniques for linear and nonlinear diffusions, nonlinar kinetic models and their scaling limits leading to macroscopic equations
Funded projects *(max. 3)*	DEASE: “Differential Equations with Applications in Science and Engineering”, Marie Curie Action Network (Responsible Project Scientist), 2006-2010 FFG Project “Analysis of Digital Images with PDE methods”, (Project leader), 2005-2006 FWF Project “Fokker-Planck and Mean-Field Equations” (Project Leader), 2000-2003
Key publications *(max. 5)*	J. Dolbeault, P.A.Markowich, D.Oelz, Ch.Schmeiser, Nonlinear Difusions as Limit of kinetic Equations with Relaxation Collision Kernels, to appear in Archives for Rational Mechanics and Analysis A. Arnold, P. A. Markowich, G. Toscani and A. Unterreiter, On Convex Sobolev Inequalities and the Rate of Convergence to Equilibrium for Fokker-Planck Type Equations, Comm. PDE, Vol. 26/1-2, 43-100, 2001 P. Gerard, P. A. Markowich, N. J. Mauser and F. Poupaud, Homogenization Limits and Wigner Transforms, Comm. Pure and Appl. Math., Vol. 50, pp. 323-378, 1997 P. A. Markowich, Applied Partial Differential Equations: A Visual Approach, Springer Verlag Heidelberg, 2006 J. A. Carillo, C. Lederman, P. A. Markowich and G. Toscani, Poincare Inequalities for Linearization of Very Fast Diffusion Equations, Nonlinearity, Vol. 15, No. 3, pp. 565-580, 2002

Other output: patents, etc. *(max. 3)*	
International network *(max. 5 names + institutions)*	Prof. Luis Caffarelli, Department of Mathematics, Univ of Texas at Austin, USA Prof. Giuseppe Toscani, Dipartimento di Matematica, Universita di Pavia, Italy Prof. Benoit Perthame, Department de Mathematics et Applications. Ecole Normale Superieure, Paris, France Prof. Stan Osher, Department of Mathematics, UCLA, USA Prof. Jose A. Carrillo, ICREA and Departament de Matematiques, Universitat Autonoma de Barcelona, Spain
Position within home institution	o. Univ.-Prof.
Size of group (No. of scientific personnel)	7
Legal form and size (No. of scientific personnel) of home institution	Faculty 63
Employment: • full time equivalent (p.a.): • salary plus employer's social security contributions (p.a.): • financed by:	 15% project contribution € 15.200.- annual project contribution University of Vienna

Track record and profile of partner 1: Univ.-Prof.DI.Mag.Wolfgang Baatz, Department of Conservation-Restauration, Academy of Fine Arts, Vienna *(please fill in the name)*

Responsibilities within the project	Artistic and technical evaluation of virtual image and fresco restoration methods, management of interaction of mathematicians and restauration professionals.
Specific competencies for the project	Competencies in the artistic field of conservation and restoration of works of art as well as in the scientific/technical field (MA in Chemistry and in Arts), about 30 years of experience in restoration
Funded projects *(max. 3)*	Restoration and preservation project of Palais Epstein, Vienna (2001-2005), Restoration of medieval wall paintings, Bishop's chapel in Gurk,

	Carinthia (2005-….); Restoration of medieval wall paintings (from the 12th century) in the church of Winkl, Lower Austria (2004-2005)
Key publications and other output *(max. 5)*	W. Baatz, Verschmutzung und Grenzbereiche, in: Schmutz – Zeitdokument oder Schadensbild?, Mitteilungen des ÖRV Band /2000, 14.Tagung des ÖRV 14.-15.6.1999, Wien, 2000 W.Baatz, G.Marian, C.Riff-Podgorschek, R.Woldron, Die neu entdeckten romanischen Wandmalereien in der Filialkirche Hl.Nikolaus in Winkl, Unsere Heimat, Zeitschrift für Landeskunde von NÖ., Jg.75 Heft 1 2004 p.63 W. Baatz, Konzepte und Methodologie zu Retusche und Ergänzung 19.Tagung des Österreichischen Restauratorenverbandes ("Mehr Schein als Sein?", Retusche – Ergänzung – Rekonstruktion – Illusion), Festspielhaus St. Pölten, NÖ, in: Mitteilungen des ÖRV, Bd.10 / 2005 W. Baatz, C. Riff-Podgorschek, Die Restaurierung der Innenräume des Palais Epstein, in: Das Palais Epstein. Geschichte, Restaurierung, Umbau, HFP ed., Löcker, Wien, 2005 W. Baatz, C. Riff-Podgorschek, Die Restaurierung der Fassade des Palais Epstein, in: Das Palais Epstein. Geschichte, Restaurierung, Umbau, HFP ed., Löcker, Wien, 2005
Size of group (No of scientific personnel)	4
Employment: • full time equivalent (p.a.): • salary plus employer's social security contributions (p.a.): • financed by:	 10% project projection € 9.300,- annual project contribution Academy of Fine Arts, Vienna

Track record and profile of partner 2: Univ.-Prof. Mag.art Brigitte Kowanz, Department of Transmedia Art, University of Applied Arts, Vienna *(please fill in the name)*

Responsibilities within the project	Artistic guidance and formulation for requirements of image processing in the medial arts
Specific competencies for the project	International media artist with focus on light-installations, interventions in architecture, environments based on time and space: www.kowanz.com Numerous national and international solo-& group-exhibitions since 1997: Professorship for Transmedia Art at the Vienna University of Applied Arts
Funded projects *(max. 3)*	In Vivo in Vitro, Permanent Installation at the Max-Planck-Institute, Münster, 2006

	"MIB - Mensch in Bewegung", Austrian Ministry of Transport, Innovation and Technology, Austrian Automobile Club (ÖAMTC), Technical Museum Vienna, 2004 "Bahnhof in Transition", Austrian Federal Railways, Vienna 2001/2002
Key publications and other output *(max. 5)*	LDM, Light_Darkness_Movements, Brigitte Kowanz, Tangente 27, Galerie der JENNOPTIK AG, Betr: Christa Häusler, Matthias Herrmann, Sabine B. Vogel, Alexandra SuessJena 2004, 2002, ISBN 3-9809285-1-9, Another Time Another Place, Brigitte Kowanz; Hrsg. Wolfgang Häusler, Beitr: Christa Häusler, Jan Tabor, Sabine B. Vogel, München 2002, ISBN 3-98088494-0-6, Timelight / Lightspace,, Brigitte Kowanz, Hrsg. Wolfgang Häusler, Betr.: Rainer Fuchs, Hatja-Cantz, Ostfildern-Ruit, ISBN 3-7757-9108, L:I.W.M.S.-Licht ist was man sieht, Brigitte Kowanz, Beitr.: von Rainer Fuchs, Rainer Metzger, Triton Verlag, 1997, ISBN 3-901310-77-0 Brigitte Kowanz, Katalog zur Ausstellung, Wiener Secession, Betr.: von Ecke Bonk, Christian Kravagna, Sabine B. Vogel, Wiener Secession, 1993, 3-900803-63-3
Size of group (No of scientific personnel)	Staff at the department: 4 Working for the project: 2
Employment: • full time equivalent (p.a.): • salary plus employer's social security contributions (p.a.): • financed by:	10% € 10.100,- University of Applied Arts

Track record and profile of partner 3: Dr.Massimo Fornasier, RICAM (Johann Radon Institute of Applied Mathematics) Austrian Academy of Sciences, Linz *(please fill in the name)*

Responsibilities within the project	Variational methods and numerical tools for image inpainting and restoration
Specific competencies for the project	Harmonic analysis, time-frequency and wavelet methods, redundant decompositions and functional analysis, variational calculus, numerical solution of PDE, digital signal and image processing, engineering and software applications for the conservation of cultural heritage (art fresco restoration).

Funded projects *(max. 3)*	2004 Individual Marie Curie Fellowship (contract MEIF-CT-2004-501018, 2 years) of the European Commission (6th Framework Programme), project title ``Flexible Time-Frequency Decompositions and Adaptive Treatment of Operator Equations by Frames''.  2003 ``Wavelets and frames in approximation theory'' (4 years), University of Rome ``La Sapienza'', Italy.  2001, Research Grant (3 years) of the Fondazione Cassa di Risparmio di Padova e Rovigo (a Bank Foundation in Padua) for the development of the ``Mantegna Project'', Italy.
Key publications and other output *(max. 5)*	S. Dahlke, M. Fornasier, T. Raasch, R. Stevenson, M. Werner: "Adaptive frame methods for elliptic operator equations: the steepest descent approach", submitted, February 2006. S. Dahlke, M. Fornasier, T. Raasch: "Adaptive frame methods for elliptic operator equations", to appear in Adv. Comp. Math., 2006 M. Fornasier: "Nonlinear projection recovery in digital inpainting for color image restoration", to appear in J. Math. Imaging Vis. 2006. M. Fornasier, D. Toniolo: "Fast, robust, and efficient 2D pattern recognition for re-assembling fragmented digital images", Pattern Recognition, Vol. 38, No. 11, 2074-2087 2005. M. Fornasier and D. Toniolo: "Computer-based recomposition of the frescoes in the Ovetari Chapel in the Church of the Eremitani in Padua. Methodology and initial results", (Italian/English), in "Mantegna nella chiesa degli Eremitani a Padova. Il recupero possibile" Ed. Skira, pp. 15-23, May 2003.
Size of group (No of scientific personnel)	2
Employment: • full time equivalent (p.a.): • salary plus employer's social security contributions (p.a.): • financed by:	 20% € 11.600,- annual project contribution Austrian Academy of Sciences

Short profile of further key researchers and project collaborators (about 5 lines per person)

Name	Competencies and responsibilities	Employment: • full time equivalent (p.a.) • salary plus employer's

		social security contributions (p.a.) • financed by:
Univ.-Ass. Dr. Norayr Matevosyan University of Vienna	Research on: Free Boundary Problems and level set methods Regularity Theory Full nonlinear Eliptic and Parabolic Equations, Photographic testing	15 % € 9.000,- annual project contribution University of Vienna
Dr.Arjan Kuijper RICAM & University of Vienna	Linear and nonlinear diffusion for Image Processing, Interaction with Prof. Kowanz	15% € 8.700.- annual project contribution, split: 50% RICAM, Linz 50% External Funding
Mag.Art Renata Burszan Academy of Fine Arts	Conservation of wall paintings	20% € 6.400,- annual project contribution by the Academy of Fine Arts
Mag.Carola Schönlieb University of Vienna	Image inpainting by higher order nonlinear Partial Differential Equations (for example Cahn- Hilliard Equations), interaction with Prof. Baatz	25% € 7.500,- annual project contribution;External Funding 25% € 7.500,- annual WWTF-contribution for 4 years
Mag. Nikolaus Passath University of Applied Arts	New media artist: applied research on realtime image- & videomanipulation; software-tracking bzw. patternrecognition (pure-data/gem/max-msp), communication with project mathematicians	50% € 7.537,50,- WWTF-contribution for 6 months
Mag. Susanne Niederberger University of Applied Arts	New media artist: artistic research on realtime images, aesthetic and semiotic aspects of programming	20% € 6.030,- annual project contribution, University for Applied Arts;

		50% € 15.075,-- annual WWTF-contribution for 4 years
Mag. Franz Schubert University of Applied Arts	New media artist: artistic research on virtual environments; 2D-Motion Graphics	50% € 7.537,50.- WWTF-contribution for 6 months
Michael Kostner University of Applied Arts	Research on aesthetic software-solutions in the artistic context, cooperations with mathematicians	50% € 12.890,- annual WWTF-contribution for one years
N.N.1 Academy of Fine Arts	Specialist in fresco restoration and conservation	100% WWTF-contribution for 1 year € 51.570,-
N.N. 2 RICAM	Post Doc, specialised in numerics for variational problems and nonlinear diffusion equations, interaction with Kowanz' group	100% WWTF-contribution for 18 months € 77.355,-
Ms. Mag. Gonca Aky University of Vienna	Ph.D student, research on free boundary problems in image processing	100% WWTF-contribution for 18 months € 45.225,-
N.N. 3 University of Vienna	Post Doc, specialised in mathematical algorithms for digital image processing and analysis,communication between mathematicians and project artists	100 % WWTF-contribution for 1 year € 51.570,-
Dr. Renate Feikes	Project Administration	12 % WWTF-contribution for 4 years € 28.000,-

-> Please remember to fill in the table "Personnel"

Optional: Comments on project structure and organisational issues

This project is highly interdisciplinary, involving Applied Mathematics, Numerical Analysis and Arts. A close collaboration of project mathematicians and project artists, which is based on regular consultations in order to develop a "common language", is planned.

We anticipate to hold joint seminars, small schools and workshops to foster the cooperation.

Also we plan to interact on a regular basis with independent artists (mainly photographers) and commercial companies producing image processing software and hardware.

Human resources development and support of women

We plan to support 10 scientific staff members on the level of pre and post docs in the 4 years project period, financed at least partly by the project. These researchers will interact with our r4esrach/artistic groups and with the pool of the ViennaCreative Industries, particularly with internationally renowned photographers and digital image retouching experts. These positions will be of the highest possible academic level, newly created in the City of Vienna by the project funds.

Handicapped applicants and females will be given preference by the recruiting procedure when comparable qualifications exist.

We expect two Ph.D students (both female), funded by the project, to graduate on topics of this project.

We remark that a female professor/artist is group leader in this project, that female research/artistic staff are employed in 3 of the 4 project teams and that the photographers involved in the project as external consultants are female.

2. Project

2.1 General Framework

2.1.1 State of the art and scientific challenge

In the last 20 years there has been an explosion of interest in variational calculus, methods of partial differential equations and their applications in image and signal processing. Since the celebrated and pioneering work by Mumford and Shah in image segmentation [15] a huge number of papers appeared on this subject and we refer to the monograph [2] for an extended (although still incomplete) list of references. Meanwhile, mathematical image processing techniques have been successfully used in medical imaging, often yielding spectacular improvements in diagnostic techniques. Moreover, in the past ten years, digital photography has been booming, and thus a significant new challenge has been raised to the mathematical image analysis/processing community: to develop tools, which allow still photographers and other visual artists to work on their digital images. Also in this realm significant progress has been made, as far as commercially available software (e.g. the market leader Adobe Photoshop) is concerned. High powered mathematical tools are nowadays at the disposition of the photographic and visual arts community, which allow for professional imaging results. However, there are many loose ends and there is a lot of room for improvement, in particular in the realm of artistic digital imaging. This project focusses on research in the restoration and creation of artistic artwork (in the form of digital images). For this work, as in many cases in digital image processing/analysis, improvement and progress can only happen through serious interdisciplinary research. Thus, it is a major challenge of this project to bring together mathematicians and artists, to define and solve new artistic challenges (in fresco restoration, creation of diffused images of light installations, quality improvement of high resolution digital images) and to use and develop new high powered mathematical tools from the field of nonlinear partial differential equations (fourth order nonlinear diffusions, new inpainting functionals, analysis of image segmentation by free boundary techniques), useful for professional applications in image restoration, photography and in the creation of new artistic works.

2.1.2 References

References

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2.2 Research questions and objectives of the project

Our work will address main applications in *art image restoration*, in particular in wall fresco restoration, and in the *creative generation* of new artistic works. Our interdisciplinary approach combines sophisticated and newly developed contemporary mathematics for the processing and analysis of digital images, based on variational calculus and nonlinear partial differential equations, with the expertise&creativity of art professionals in image restoration and in creative art. In particular we shall develop numerical techniques and software for the virtual restoration of damaged or incomplete wall frescos and for the creative design of new small and large scale light installations. This work shall give the visual artists new tools which support and enhance their creativity, based on a development of new mathematical tools. It will be carried out in tight cooperation of top-level mathematicians and art professionals, mainly working and living in Vienna.

2.3 Work plan

General overview

Art image Restoration (Work packages 1,2 and 4). Aside from free discontinuity formulations [15], models based on total variation (TV) minimization have been proposed for digital image restoration. In their seminal work [16], Rudin, Osher, and Fatemi proposed the denoising model

$$\arg\inf_u \left\{ F(u) = \int_\Omega |\nabla u| + \mu |f-u|^2 \right\}. \tag{1}$$

Several extensions and modifications of this model have been recently introduced [6]. In particular, the simple total variation inpainting model [5] is just a slight modification of (1) and it reads as follows

$$\arg\inf_u \left\{ F(u) = \int_\Omega |\nabla u| + \mu \int_{\Omega\setminus D} |f-u|^2 \right\}. \tag{2}$$

Here, one seeks an image u that coincides with f on $\Omega \setminus D$, which is the part of the domain where the image is assumed to be given correctly, and it extends f into D, the occlusion area, where the image is considered distorted or even totally missing.
Second order variational inpainting methods, like total variation inpainting, have drawbacks, for example in the connection of edges over large distances or the continuous propagation of level lines into the damaged domain. In an attempt to solve both the connectivity principle and the so called staircase effect resulting from second order image diffusions, a number of third and fourth order diffusions have been suggested for image inpainting.
One of the most important works in this direction is the algorithm of Chan, Kang and Shen [7] based on Euler's elastic energy. Their approach leads to a continuous connection of level lines also over large distances, compare Figure 2. Another new approach in the class of fourth order inpainting algorithms is inpainting of binary images using the Cahn-Hilliard equation [3].
In image restoration, automatic detection of inpainting regions is a crucial task. Image segmentation [2, 6, 15] is the partition of a digital image into multiple regions (sets of pixels), according to some criterion (e.g., discontinuity of colors). The goal of segmentation is typically to detect the edges in the given image and locate certain objects of interest. Segmentation could therefore be seen as a computer vision problem.
We shall approach the modeling of multivariate and vector valued problems by methods of variational calculus based on total variation minimization and free discontinuity models, and by high order PDE methods. In particular, we shall consider applications of image inpainting for digital color images, resolution restoration and deblurring/denoising of digital color images, and image segmentation. With these general image processing tools, we shall address realistic problems in art restoration.

Creative art generation (Work package 3). TV (total variation) methods seek a solution that consists of piece-wise flat cartoon-like regions. Doing so, one typically finds segments clearer than those obtained by determining zero-crossings of a Laplacian filter, a method that was common in the early years of image analysis and can be implemented with a simple PDE describing heat flow, see e.g. [14]. Comparing the results with human observers, one finds that TV methods do not necessarily yield *better* results. The human visual system is, for instance, capable of "looking through noise", that is, we are more sensitive to low-pass than high-pass filtering.

We shall mimic this human ability by investigating two types of diffusion based PDEs, which incorporate the aforementioned methods. These PDEs follow from energy minimization arguments, as well as from geometrical observations. They are geometrically motivated evolutions described by certian combinations of derivatives in the directions tangent and normal to level lines of an image: p-Laplacians and weighted combinations of second order gauge coordinate derivatives. Theoretical and perceptual aspects will be investigated. Their characteristic behavior will be employed conceptually to the modification and creation of modern visual artistic works.

Work package 1: *Variational inpainting of color images with cross-channel and additional-channel constraints.*

Description and methods

Inspired by the art fresco restoration problem in [11, 10] (see `http://www.pd.infn.it/~labmante`) ('Mantegna Project'), we presented in [9] the following model:

$$\arg\inf \left\{ F(u) = \mu \int_{\Omega \setminus D} |u(x) - f(x)|^2 dx + \lambda \int_D |\mathcal{L}(u(x)) - f(x)|^2 dx + \int_\Omega \phi(|\nabla u(x)|) dx, u \in W^{1,2}(\Omega, \mathbb{R}^M_+) \right\}, \tag{3}$$

where f is the given observed multiple channel image, which is presumed to be colored on $\Omega \setminus D$ (i.e., where the fragments are located) and only partially given on D (for example, only gray or ultraviolet levels are given there). The projection from RGB+additional channels to lower dimensional information is given by the (nonconvex and nonlinear) function $\mathcal{L}$. Such a model can be viewed as a generalization of the more classical *inpainting/disocclusion problem* previously recalled (with $\lambda = 0$), see also [5].

The functional in (3) admits minimizers in BV in the sense that there exists $u \in BV(\Omega; \mathbb{R}^M)$ solving the problem:

$$\begin{aligned} \min\{\overline{F}(v) &= \mu \int_{\Omega \setminus D} |v(x) - f(x)|^p dx + \lambda \int_D |\mathcal{L}(v(x)) - f(x)|^p dx \\ &+ \int_\Omega \sum_{i=1}^M \phi(|\nabla v_i|) dx + c \left(\sum_{i=1}^M \int_{\Omega \setminus S_{v_i}} |C_{v_i}| + \int_{S_{v_i}} (v_i^+ - v_i^-) d\mathcal{H}^{d-1} \right) \}, \end{aligned}$$

where C_v is a measure supported on a Cantor set, S_v is the approximate discontinuity set of v, and $\mathcal{H}^{d-1}$ is the $(d-1)$-dimensional Hausdorff measure. In particular it is

$$\overline{F}(u) = \inf_{v \in W^{1,2}(\Omega, \mathbb{R}^M)} F(v).$$

The interplay between functional and geometrical aspects is stronger for so-called special bounded variation functions, whose space is denoted by $SBV(\Omega)$ [8]. In order for the solution image to be in SBV, one has to formulate the model as a free discontinuity problem. Therefore instead of (3) one can minimize

$$F(u, \Gamma) = \alpha \int_{\Omega \setminus D} |u(x) - f(x)|^p dx + \beta \int_D |\mathcal{L}(u(x)) - f(x)|^p dx + \int_{\Omega \setminus \Gamma} |\nabla u|^2 dx + \gamma \mathcal{H}^{d-1}(\Gamma \cap \Omega),$$

in the set

$$\mathcal{A} := \{(\Gamma, u) : \Gamma \subset \overline{\Omega} \text{ closed }, u \in W^{1,2}_{loc}(\Omega \setminus \Gamma; \mathbb{R}^M)\}.$$

For $u = (u_1, ..., u_M)$, a M-channel signal, one assumes (as for example for color images) that $\Gamma = \cup_{i=1}^M \Gamma_i$, being Γ_i the discontinuity set of u_i. In the paper [4] by Brook et al., it has been pointed out that the use of the smoothing term $\int_{\Omega \setminus \Gamma} |\nabla u|^2 dx$ in the vector valued color images may induce certain artifacts around discontinuities due to the fact that the channels do not correctly couple. Brook et al. suggest possible variations where this energy is replaced by $\int_{\Omega \setminus \Gamma} G(\nabla u) dx$, where G is a suitable polyconvex function. This modification imposes that different channels cannot have large derivative components in different directions and therefore they correctly couple at discontinuities. Still the channels are allowed to be uncoupled on zero order terms, since they may represent independent components (e.g., different frequencies such as color and ultraviolet spectra). The existence of minimizers in $SBV(\Omega;\mathbb{R}^M)$ of such functionals will be approached by adapting the techniques of [12].

Expected results

Besides the functional modeling and the theoretical analysis of such models, one wants to have also efficient numerical methods in order to carry out simulation runs. In fact, the ultimate goal of this work package is the development of an innovative software tool for the computer assisted restoration/inpainting of visual art works, and in particular of damaged frescoes, see e.g. Figure 1. The mathematical modeling and the corresponding implementation are expected to estimate and to predict parametrically morphological and geometrical features of the missing parts from the known data on existing undamaged image parts.

Figure 1: The Neidhart frescoes are the oldest non-religious wall paintings in Vienna and date back to the 14th century. Painted for the wealthy Viennese merchant Michel Menschein, the work is believed to have started in 1398. The frescoes recount scenes from the songs and poetry of Neidhart von Reuental, a court minstrel and social commentator who lived during the first half of the 13th century. The photographs are courtesy of Andrea Baczynski.

The approach we shall develop is the approximation of the target functionals by Γ-converging quadratic functionals in $W^{1,2}(\Omega;\mathbb{R}^M)$ [13]. The computation of minimizers of the latter can be done by solving the associated variational elliptic PDEs.

Milestones

M1: Analytic study of the existence of minimizers of the proposed functionals. Investigation on the uniqueness of solutions;
M2: Γ-convergence approximation and associated Euler-Lagrange equations;
M3: Numerical schemes for the minimization of the functionals;
M4: Software development and ongoing interaction of mathematicians and art restoration professionals;
M16: Fresco restorations aided by simulation results;
M18: Virtual "Tratteggio" retouching algorithm.

Work package 2: *Higher order flows in inpainting.*

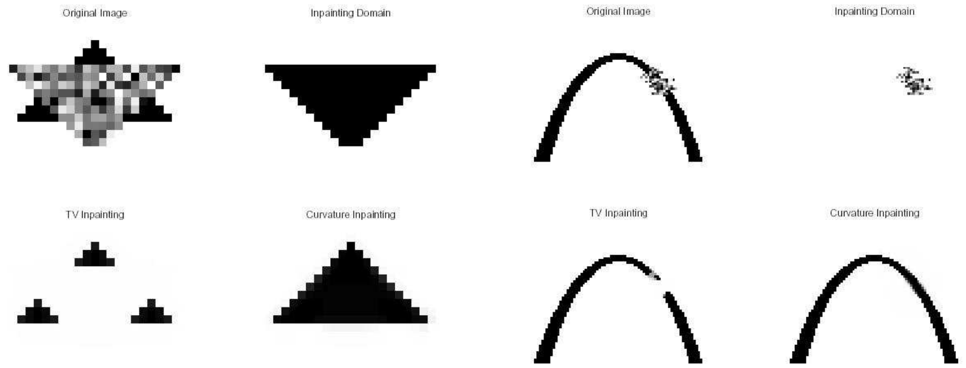

Figure 2: Two examples of elastica inpainting compared with TV inpainting. In the case of large aspect ratios the TV inpainting fails to comply to the Connectivity Principle. Figure from [7]

Description and methods

Let f be a given image in a domain Ω with inpainting domain $D \subset \Omega$. The result of the Cahn-Hilliard inpainting approach u evolves in time to become a fully inpainted version of f, under the following fourth order partial differential equation

$$\frac{\partial u}{\partial t} = \Delta(-\epsilon \Delta u + \frac{1}{\epsilon}\mathcal{L}'(u)) + \lambda(f - u),$$

with $\mathcal{L}$ is a nonconvex nonlinearity and

$$\lambda = \begin{cases} \lambda_0 & \Omega \setminus D \\ 0 & D. \end{cases}$$

The main challenge in inpainting with higher order flows is the detailed analytic analysis of third and fourth order approaches and their effective numerical implementation.
Existence and uniqueness results and detailed knowledge of regularity and stability of solutions are crucial for forecasting and interpreting the outcome of the inpainting algorithms. In particular, information on the regularity of solutions is important for a priori error estimates of the results of the inpainting algorithms. Besides the well-established theory of harmonic functions, a regularity analysis for the solutions of most of the inpainting methods is still missing.
In numerical discretisations the order of the equation and the occurrence of a possibly non-convex term as in the example of the Cahn-Hilliard equation require small temporal step sizes and semi-implicit or even implicit temporal discretization schemes to guarantee numerical stability. Spatial discretizations taking into account the geometry of the equation and numerical acceleration techniques such as the multigrid method and the multiresolution decomposition are important challenges.

Expected results

Higher order equations are much more difficult to analyze than second order equations and many techniques, which work in the second order case, fail badly in the fourth order case (e.g. the max-min principle). It remains to be seen whether the application of forth order equations significantly improves inpainting results, compared to the rather well understood second order equations. We expect that the application of higher order flows to restoration problems could bring better results in certain cases. Especially if bigger parts of the image are missing a preprocessing step with a higher order flow is recommendable to link the correct edges over a wide range. Also for the inpainting of fine and complex textures, higher order flows transport the structure from the boundary into the inpainting domain in a very smooth way. A detailed study of these working hypothesis will be carried out on a wide range of images (natural and artificial images, artistic works).

Milestones

M5: Analytic study of selected third and fourth order equations (existence, regularity, stability);
M6: Numerical analysis and efficient, accurate numerical implementation. Testing and interpretation of the results;
M7: Software development and ongoing interaction of mathematicians, art restoration professionals and photographers.

Work package 3: *Diffusion based image analysis.*

Description and methods

Consider the so-called p-Laplacian energy: $E = \frac{1}{p}\int_\Omega |\nabla u|^p \; d\;\Omega$, where the image intensity u is defined on an domain $\Omega \subset \mathbb{R}^2$. The variational derivative reads $\delta E = -\nabla \cdot \left(|\nabla u|^{p-2}\nabla u\right)$ and the steepest descent evolution $u_t = -\delta E$ gives the p-Laplacian PDE describing a nonlinear evolution of an initial image. The TV method ($p = 1$) and the heat equation ($p = 2$, linear case) are special instances of this general evolution. Obviously, additional constraints have to be prescribed in order to reach a solution that differs from the trivial zero-solution in the large t limit. In many cases, the continuous evolution of the original image is of interest in both constrained and unconstrained cases. Four special instances will be studied: $p < 1$, $1 < p < 2$, $p > 2$ and the limit $p \to \infty$.

Insight into the behavior of the p-Laplacian evolution of an image is obtained by using gauge coordinates [14]: We fix at each point locally the local coordinate frame in such a way that one frame vector points into the direction of maximal change of the intensity function u and the other perpendicular to it (90 degrees clockwise, say). The direction of maximal change of intensity is the gradient vector $w = (u_x, u_y)$. The perpendicular direction is $v = (u_y, -u_x)$. Taking derivatives with respect to these directions, we find $u_v = 0, u_w = \sqrt{u_x^2 + u_y^2}$. Then $u_t = -\delta E = u_w^{p-2}\left((p-1)u_{ww} + u_{vv}\right)$, with the identity $u_{ww}+u_{vv} = \Delta u$, the Laplacian. The evolution of u is completely described by a combination of the three occurring derivatives. Obviously, for large p the influence of the first order derivative is strong. Therefore, a second type of PDEs will be investigated,too: $u_t = ((1-q)u_{ww} + qu_{vv})$, with $0 \le q \le 1$, a weighted combination of the two second order derivatives. For $q = 1$ the mean curvature motion is obtained, while $q = \frac{1}{2}$ yields the heat equation. The case $q = 0$ relates to the infinity-Laplacian, representing a kind of extreme blurring [1].

Expected results

Besides theoretical insight into the yet mostly unknown behavior of images under the evolutions of the suggested class of PDEs, efficient numerical methods need to be developed for enabling their practical use. The advantage here is the possibility of analyzing and processing (denoising) images with different values of p and, resp. q, thus generating a lot of additional information out of the original image by introducing another (other than t) scale-space-type variable. Also, a generalized structure for automatically segmenting images as in [14] for the $p = 2$ case is to be sought. Moreover, the p-Laplacian evolution for large values of p appears to have the effect of blurring large structures, without completely removing noise. This mimics a feature of the human visual system, and will be

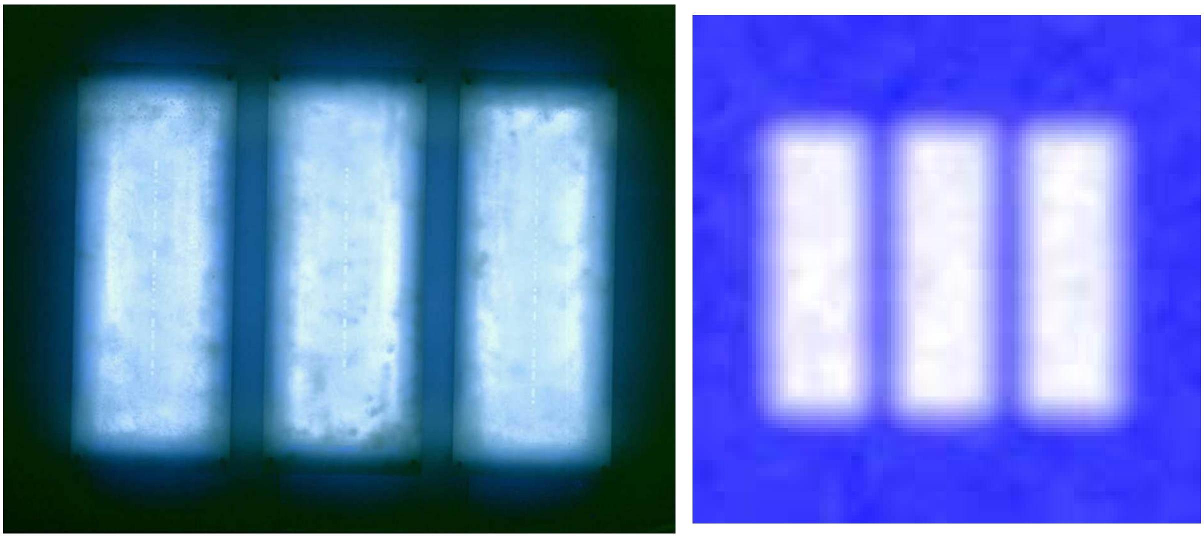

Figure 3: Left: Artwork by Prof. Brigitte Kowanz, see `http://www.kowanz.com`. Right: Experimental result of the evolution of the p-Laplacian for $p = 20$ on an artificial image with three bars and noise.

used in a conceptual and creative way by artists, see Figure 3, left, for a first example.

Milestones

M8: Literature survey and derivation of main theoretical aspects of the PDEs;
M9: algorithmic and numerical implementation of PDEs;
M10: validation of results with human perception;
M11: fine tuning of algorithms with desired human-observer-based results.
M15: Construction of light installations based on diffusively smoothed images.

Work package 4: *A two phase free boundary problem emerging from piecewise smooth Mumford-Shah segmentation in image processing.*

Description and methods

Let the image domain be $\Omega \subset \mathbb{R}^2$ and the real valued function $u_0 \in L^\infty$(for a gray scale picture) defined on Ω represent the original image intensity function. Then the classical segmentation problem consists of finding an approximation of u_0 by a simpler image u, which captures "the main features" of u_0. An effective way of finding this simplified intensity function u is the piecewise smooth Mumford-Shah segmentation by level sets (see [6] and the references within). We consider the famous Mumford-Shah functional (see [15]):

$$J(u, \Gamma) = \int_\Omega (u - u_0)^2 + \alpha \int_{\Omega \setminus \Gamma} |\nabla u|^2 + \beta H^{n-1}(\Gamma) \tag{4}$$

and look for a pair (u, Γ) that minimizes the above functional J. Here $\Gamma \subset \Omega$ is again the set of discontinuities, corresponding to the significant edges in the image. The functional penalizes deviations from the original image, deviations from (piecewise) smoothness within each region and the total length of the boundaries of all the regions (edges). The main difficulty in dealing with this problem is that one has to minimize J with respect to not only a function u, but also an apriorily unknown set Γ. Exactly this property makes it a free boundary problem. The following two phase situation arises in piecewise smooth Mumford-Shah segmentation by level sets (see for example [6] and the references within). Let the idealized image u be segmented into u^+ and u^- by the level set function ϕ:

$$u(x) = u^+(x)\chi_{\{\phi>0\}} + u^-(x)\chi_{\{\phi<0\}} \tag{5}$$

where χ_D is defined to be the characteristic function of the set D with value 1 on D and 0 outside. Substituting the above expression into (4) we obtain the following functional which acts on the image segments and on the level-set function:

$$E(u^+, u^-, \phi) = \int_\Omega (u^+\chi_{\{\phi>0\}} + u^-\chi_{\{\phi<0\}} - u_0)^2 + \tag{6}$$

$$\alpha \int_\Omega |\nabla u^+|^2 \chi_{\{\phi>0\}} + \alpha \int_\Omega |\nabla u^-|^2 \chi_{\{\phi<0\}} + \beta H^{n-1}(\Gamma), \tag{7}$$

where $\Gamma = \{\phi = 0\}$ is the 0-level set. The above problem differs from "standard" free boundary problems as for example discussed in [18], [17] by the fact that the characteristic functions occurring under the integrals do not depend on the unknown u itself, but on the level set function ϕ. This is non-standard in free boundary theory, and the main reason for the mathematical difficulties occurring in the analysis of the minimization problem.

Expected results

We shall study the optimal regularity of the minimizing functions u^+ and u^- as well as the local regularity of the free boundary Γ. To achieve this, we shall adopt and enhance the techniques recently developed for a similar class of free boundary problems (see for example [18], [17]). This is a new way of interpreting a free boundary problem, namely we exchange the role of the geometric unknown Γ by the unknown level set function ϕ. Thus, instead of searching for the solution of the free boundary problem as a pair (u, Γ), we shall search for $(u^\pm, \phi)$. This will give the benefit of applying well developed free boundary techniques to gain deep insights into the theory of image segmentation.

Milestones

M12: Investigate the growth of the solutions, prove non-degeneracy and find the optimal regularity of the solutions;
M13: Using the results from M12, classify the global solutions of the problem;
M14: Using the classification of the global solutions, prove the local optimal regularity of the free boundary;
M17: Retouching of photographic images.

2.4 Relevance to the creative industries and to Vienna

Art image restoration. The Neidhart frescoes (Tuchlauben 19, 1010 Vienna) are a perfect example of a damaged art work where computer assisted restoration can be fruitfully applied. In intensive cooperation with o.Univ.Prof.Mag.DI Wolfgang Baatz (Akademie der bildenden Künste Wien), we intend to develop software for the automatic inpainting/re-touching of the missing/ruined part of the frescoes. A flexible choice of parameters in the software will allow to follow several possible restoration hypotheses which can be evaluated by the art professional prior to the real-life restoration. Also for those works for which a real restoration is considered dangerous or not immediately advisable, the virtual restoration can anyway be used to experiment with possible reconstructions and to show results to the public, without the risk of compromising the integrity of the work. It is worth to mention that the case of the Neidhart frescoes is not isolated in Vienna, and we expect that our software will be fruitfully implemented for other art works in Austria and abroad (for example in Italy).

Creative art generation. The nonlinear evolutionary diffusive PDEs, as described in Work package 3, can be tuned to yield either cartoon-like images, or blurred ones. In the former case, real-life images can be transformed to cartoons. The latter, as shown in an exemplary way in Figure 3, relates to modern artistic view and human perception of structures. In cooperation with Univ.Prof. Mag. art. Brigitte Kowanz (Universität für angewandte Kunst, Wien), we intend to develop software based on nonlinear diffusions for new applications in contemporary visual art. In particular, we shall investigate the visual results of nonlinear diffusive mathematical distortion/deformation of art images of large and small scale light installations, and the use of this tool for creative applications. The blurring of large structures in the case of p-Laplacian evolution for $p > 2$ can be concretely exploited and practically reproduced for decorative purposes in large environments. Brigitte Kowanz uses the properties of reflective materials in large-format objects and installations, where materiality appears entirely dissolved in the play of reflection and painting, transformed at the moment of its encounter with light into an auratic image. We expect this creative process to benefit significantly from the digital image processing (diffusive) tools developed in this project.

Moreover, we shall work with Vienna based photographers, like Gabriele Rothemann (artistic photography), Anna Blau (architectural photography) and Margherita Spiluttini (www.spiluttini.com) to get their feedback and help them with their demands on image processing. We believe that the techniques developed in this project, particularly the image smoothing and segmentation methods, have a rather direct application in improving high resolution digitally captured or high resolution and analysing scans of analog images. We plan to share our insights with the imaging professionals of the 'Digital Store Vienna' (http://www.digitalstore.at/) and at 'Foto Digital Loho' (http://lo000040.host.inode.at/) through continuous consultations and seminars. Excellent working contacts exist already and will be enhanced through the project work.

2.5 Characterization of multidisciplinary approaches

Interdisciplinary scientific/research projects are generally more difficult to execute than intradisciplinary ones, but, in our view, much more rewarding. Already projects involving artists of different disciplines or mathematicians and physicists have inherent difficulties and pitfalls. Much more so, a collaborative project between artists and mathematicians is a great challenge for everybody involved and definitely poses a serious commitment to the participants, which cannot be taken ligthly. However, we believe that our group of project proposers has a very good chance to succeed, on a scientific, artistic and eventually also on an economic level.

We started from common interests and had an intensive project preparation phase in which a common scientific/artistic ground was established and basic methodology was agreed upon. The communication of the mathematicians in the project (Markowich, Fornasier) with our image conservation and restoration specialist (Baatz), who also happens to hold a university degree in chemistry, turned out to be rather straightforward on scientific grounds while the communication with our medial artist (Kowanz) is based on more intuitive grounds of fascination with geometry, formal aesthetics and her use of light as a transportation agent of information. Although interdisciplinarity is not a goal in itself, we believe that one of the benefits of this project will be to lay the groundwork for a fruitful professional dialog between art and mathematics, which to our knowledge is so far totally lacking in Austria and scarce even on the international level.

Modern digital image processing requires complicated mathematical tools, in most cases far beyond the scope and knowledge base of imaging artists, and in many cases also beyond the possibilities of commercially available software (in spite of its great sophistication). On the other hand, the requirements of modern visual artists pose an important challenge to the mathematical image processing community. Clearly, both communities can profit a lot from interdisciplinary interaction.

This project focusses on direct interaction of artists and mathematicians in the area of imaging science/visual arts. The mathematicians in the project shall develop new image processing tools by proceeding to satisfy the artists' demands in applications concerned with fresco restoration, virtual creation of visual artistic works by image processing techniques and in processing and improving high resolution digital photographies. In turn the project artists will push forward their own art by challenging the mathematicians to develop new image processing techniques in the areas of partial differential equations and variational analysis.

We plan on holding regular joint seminars and field trips. Also, it is planned that scientific-mathematical staff financed by this project shall spend a nontrivial portion of the working time with the groups of the project artists and that group members of the project artists shall meet for regular consultations with the mathematicians.

2.6 Prospective benefits

We believe that this cooperation between artists and mathematicians will have excellent medium and long term economic benefits. Art works sell on the national and international market (which is thriving) and are shown in public and private galleries and museums. Our project strives to give artists new tools to improve their performance. The commercial implications of improving output of photographers and of other visual artists (like medial artists dealing with light installations) are evident, and virtually restored frescos can be shown on an interactive level to the general public.

Eventually, we also expect economic benefits from licensing image processing and segmenting software for high resolution digital images, particularly in the areas of creative and non-damaging image sharpening and denoising, for the photographic community.

3. Planning of Costs and Finance *(about 2-3 pages)*

Optional: General comments on cost structure

Although we know that € 500.000,-is a soft upper bound for the funding of WWTF-projects, the grand total of this project amounts to approximately € 577.000,- . We remark that this is a very large, interdisciplinary project with 4 involved institutions of very different orientations and we believe that due to the proposed scientific/artistic deliverables of our project, the requested increase in total funding is justified.

-> ***Please remember to fill in the table "Costs&Finance".***

Description of material expenses **to be funded** at *project managers home institution*

Special equipment /devices (name, purchase price, life time and deprecia-tion rates during project time)	2 Workstations with colour management enabled displays (for example EIZO), € 8.000,- each, 4 years life time, depreciation during the project time: 100 %; 1 digital reflex camera with 3 lenses, € 10.000-, 4 years life time, depreciation during the project time: 100 %; 1 notebook computer, € 3.500, 4 years life time, depreciation during the project time: 100 %
Materials/software	
Other expenses	€ 20.000,- Travel expenses
Overheads	€ 50.000,-

Description of material expenses **to be funded** at *other institutions*

Special equipment /devices (name, purchase price, life time and deprecia-tion rates during project time)	Academy of Fine Arts: 3 trolley luminaires, € 8.000,- 4 years lifetime, depreciation during the project 100%; 1 notebook computers, € 3.500.- , 4 years lifetime, depreciation during the project time 100%; 1 Linhof Technikardan 6x9 cm medium format view camera with 2 le-nes, € 10.000,- , 4 years lifetime, depreciation during the project time 100%; 1 Workstation € 3.000,- 4 years lifetime, depreciation during the project time 100 % University of Applied Arts: 2 Workstations, € 4.000,- each, 4 years lifetime, depreciation during the project time: 100 % 2 notebook computers, € 3.500,. each, 4 years lifetime, depreciation during the project time 100%

	RICAM: 1 Workstation € 4.000,- 4 years lifetime, depreciation during the project time 100% 1 notebook computer, € 3.500,- 4 years lifetime, depreciation during the project time 100%
Materials/software	
Other expenses	University of Applied Arts € 10.000,- travel expenses RICAM € 10.000,- travel expenses
Overheads	€ 35.000,-

Description of **contribution** by *project managers' home institution*

Special equipment /devices (name, purchase price, life time and deprecia-tion rates during project time)	1 Workstation, € 3.000,- depreciation 100 % during the project period.
Materials/software	
Others (cash, overheads, etc.)	

Description of **contribution** by *other institutions*

Special equipment /devices (name, purchase price, life time and deprecia-tion rates during project time)	Academy of Fine Arts: Worksite equipment for conservation and retouching, studio infrastruc-ture, € 5.000 per year, no depreciation RICAM: 1 Workstation, € 3.000,- 4 years lifetime, depreciation during the pro-ject time 100%
Materials/software	
Others (cash, overheads, etc.)	

Disclosure of other applications for funding

Has the project (or parts of it) already been subject to any other requests for funding?

No other funding requests

Appendix

3 Tables *(Excel file "Proposal_Tables.xls")*

CVs

LOIs